

The Distances among the Particles in the Space

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<https://emfps.blogspot.com/2018/04/the-distances-among-particles-in-space.html>

In physics and engineering, the distances among the particles is usually an important factor for analysis of the subject especially when we are studying Newton's law of universal gravitation in mechanic, Coulomb's law in electricity, Maxwell's equations in electromagnetic theory and so on. But, do you think that the application of the theorems in related to the distances among the particles is limited to above laws in classic physics? The most crucial thing is to predict a flexible system which is moving. For instance, suppose you have a bowl filled by water in your hand. When you move, how can you predict the motion of water system into the bowl? Of course, when we are speaking about a rigid system, the prediction and analysis of this system is easy just like when you are moving by a car (an approximately rigid system).

In the reference with article of "A Specific Permutation And Applications (1)"

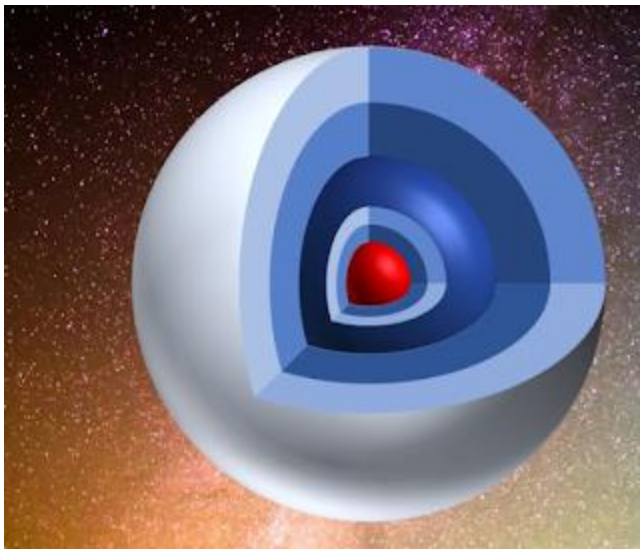
(<http://www.emfps.org/2018/03/a-specific-permutation.html>), we can derive many conjectures (or theorems) or properties in related to the distances among the particles in the space.

Consequently, this could be conducted as a big project in which I will not be able to handle this project alone.

In this series, I will post little by little these new properties accompanied by examples only related to Newton's law of universal gravitation in mechanic. Since I am using the big data analysis to extract these properties, I have not any pure mathematics proof for them. Therefore, I will mark up them as the Conjectures instead of the Theorems. *As the matter of fact, this is the Big Data Analysis with Microsoft excel to create the value from data.*

Conjecture (1):

If above article (A Specific Permutation...) has been applied for only three choices from many events (sample space), we will have the concentric spheres or layered spheres just like below figure:



In previous article, I told you a specific permutation with $n = 6$ and $r = 3$, gives us 48 points on a sphere, If we increase the sample space (n) to 8 and $r = 3$, we will have 4 concentric spheres which gives us 48 points on each sphere and for $n = 10$, we will have 10 concentric spheres which gives us 48 points on each sphere and for $n = 12$, we will have 20 concentric spheres which gives us 48 points on each sphere and etc..

In fact, the number of the layered spheres can be calculated by using previous equation divided to

48.

Note: I will again comeback to this conjecture as soon as possible.

Conjecture (2):

The distance of each point on a sphere is equal to at least five pair points on the same sphere.

Suppose points $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}$ have been located on a sphere.

This conjecture says to us:

$$d(P_1, P_2) = d(P_1, P_3) = A$$

$$d(P_1, P_4) = d(P_1, P_5) = B$$

$$d(P_1, P_6) = d(P_1, P_7) = C$$

$$d(P_1, P_8) = d(P_1, P_9) = D$$

$$d(P_1, P_{10}) = d(P_1, P_{11}) = E$$

Where:

$d(P_1, P_2)$ = the distance between point P_1 to P_2

The amounts of A, B, C, D, E are the members of real number

Note: I will again comeback to this conjecture as soon as possible.

Conjecture (3):

The distances among the points $P1(x, y, z)$, $P2(z, x, y)$ and $P3(y, z, x)$ are always the same.

$$d(P1, P2) = d(P1, P3) = d(P2, P3)$$

Example:

Suppose point P1 has below coordination:

P1 (3, -5, 11)

Therefore, the coordination P1 and P2 will be:

P2 (11, 3, -5)

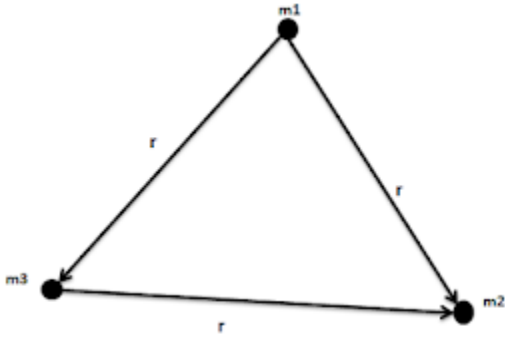
P3 (-5, 11, 3)

$$d(P1, P2) = d(P1, P3) = d(P2, P3) = 19.59592$$

Let me tell you an example about Newton's law of universal gravitation as follows:

The Mapping A System of Three Particles for Given Gravity Potential Energy And The Distance

Suppose we have three particles P1, P2 and P3 with the distances among them in infinity which have the masses of m_1 , m_2 and m_3 . If an external force brings all these particles in new location just like below figure, how can we calculate total work done on them?



It is clear; the work done on particles is equal to total gravity potential energy of system but in opposite direction in which we can write below equation for total gravity potential energy of system:

$$U(r) = -\left(\frac{G \cdot m_1 \cdot m_2}{r} + \frac{G \cdot m_1 \cdot m_3}{r} + \frac{G \cdot m_2 \cdot m_3}{r}\right)$$

Where:

G = Gravitational constant = 6.672×10^{-11} (N.m²/kg²)

$U(r)$ = total gravity potential energy of system (J)

r = the distance among the particles (m)

m_1, m_2, m_3 = mass (kg)

Question (1):

If we have:

$$U(r) = -0.04143312 \text{ J}$$

$$r = 0.0002 \text{ m}$$

How can we map these particles in the space?

Step (1):

To find masses of particles, we should apply the method mentioned in article of “Solving a Nonlinear Equation with Many Independent Variables” (<http://www.emfps.org/2016/10/can-we-solve-nonlinear-equation-with.html>)

I found 6 answers for mass of the particles:

m1	m2	m3
6	300	400
6	400	300
15	380	300
22	100	1000
40	130	700
40	330	300

Step (2):

Since the distance is equal to 0.0002 m, we can find the coordination of particle P1 by using the model presented in article of “A Model to Track the Location of a Particle in the Space” (<http://www.emfps.org/2018/02/a-model-to-track-location-of-particle.html>)

I found 408 answers for coordination of particle P1 as follows:

Here I have posted some of the results:

Models	x	y	z
1	-0.0005	0.00047	0.00035
2	-0.0005	0.00037	0.00035
3	-0.0005	0.00035	0.00047
4	-0.0005	-	-

		0.00035	0.00037
5	- 0.00047	- -0.0005	- 0.00035
6	- 0.00047	- 0.00045	- 0.00032
7	- 0.00047	- 0.00035	- -0.0005
8	- 0.00047	- 0.00035	- 0.00032
9	- 0.00047	- 0.00032	- 0.00045
10	- 0.00047	- 0.00032	- 0.00035
11	- 0.00045	- 0.00047	- 0.00032
12	- 0.00045	- 0.00042	- 0.00029
13	- 0.00045	- 0.00032	- 0.00047

14	- 0.00045	- 0.00032	- 0.00029
15	- 0.00045	- 0.00029	- 0.00042
16	- 0.00045	- 0.00029	- 0.00032
17	- 0.00042	- 0.00045	- 0.00029
18	- -	-0.0004	- -

	0.00042		0.00027
19	- 0.00042	- 0.00029	- 0.00045
20	- 0.00042	- 0.00029	- 0.00027
21	- 0.00042	- 0.00027	- -0.0004
22	- 0.00042	- 0.00027	- 0.00029
23	-0.0004	- 0.00042	- 0.00027
24	-0.0004	- 0.00037	- 0.00024
25	-0.0004	- 0.00027	- 0.00042
26	-0.0004	- 0.00027	- 0.00024
27	-0.0004	- 0.00024	- 0.00037
28	-0.0004	- 0.00024	- 0.00027
29	- 0.00037	- -0.0005	- 0.00035
30	- 0.00037	- -0.0004	- 0.00024
31	- 0.00037	- 0.00035	- -0.0005
32	- 0.00037	- 0.00035	- 0.00022

33	- 0.00037	- 0.00024	- -0.0004
34	- 0.00037	- 0.00024	- 0.00022
35	- 0.00037	- 0.00022	- 0.00035
36	- 0.00037	- 0.00022	- 0.00024
37	- 0.00035	- -0.0005	- 0.00047
38	- 0.00035	- -0.0005	- 0.00037
39	- 0.00035	- 0.00047	- -0.0005
40	- 0.00035	- 0.00047	- 0.00032
41	- 0.00035	- 0.00037	- -0.0005
42	- 0.00035	- 0.00037	- 0.00022
43	- 0.00035	- 0.00032	- 0.00047
44	- 0.00035	- 0.00032	- 0.00019
45	- 0.00035	- 0.00022	- 0.00037
46	- 0.00035	- 0.00022	- 0.00019
47	- 0.00035	- 0.00019	- 0.00032

48	- 0.00035	- 0.00019	- 0.00022
49	- 0.00032	- 0.00047	- 0.00045
50	- 0.00032	- 0.00047	- 0.00035
51	- 0.00032	- 0.00045	- 0.00047
52	- 0.00032	- 0.00045	- 0.00029
53	- 0.00032	- 0.00035	- 0.00047
54	- 0.00032	- 0.00035	- 0.00019
55	- 0.00032	- 0.00029	- 0.00045
56	- 0.00032	- 0.00029	- 0.00017
57	- 0.00032	- 0.00019	- 0.00035
58	- 0.00032	- 0.00019	- 0.00017

Please be informed that each answer gives us a model of three particles P1, P2 and P3. For instance, look at Model (1) in above table. By applying the conjecture (3), you can find below system of articles:

	x	y	z	Distance
P1	-0.0005	-0.00047	-0.00035	0.00020
P2	-0.00035	-0.0005	-0.00047	0.00020
P3	-0.00047	-0.00035	-0.0005	0.00020

As you can see, the distance among P1, P2 and P3 is equal to 0.0002 m.

Now, please comeback to step (1), we have 6 answers for the masses and for each answer of masses, we have 408 models. It means that we found 2448 systems of three particles which have the same total gravity potential energy.

Question (2):

If we have:

$$U(r) = -0.01748064 \text{ J}$$

$$r = 0.0002 \text{ m}$$

How can we map these particles in the space?

It is clear, the number of models is 408 but the answers of masses are as follows:

m1	m2	m3
4	100	500
10	240	200
20	20	1300
20	220	200
30	380	100
31	200	200
40	60	500

It means that we found 2856 systems of three particles which have the same total gravity potential energy.

Question (3):

If we have:

$$U(r) = -2.27021E-06 \text{ J}$$

$$r = 1.54 \text{ m}$$

How can we map these particles in the space?

The answers for masses are just like to question (2):

m1	m2	m3
4	100	500
10	240	200
20	20	1300
20	220	200
30	380	100
31	200	200
40	60	500

But the answers for step (2) are as follows:

Models	x	y	z
1	-0.31	0.09	0.923333
2	-0.31	0.523333	0.923333
3	-0.31	0.923333	0.09
4	-0.31	0.923333	0.523333
5	-0.27667	0.123333	0.956667
6	-0.27667	0.556667	0.956667
7	-0.27667	0.956667	0.123333
8	-0.27667	0.956667	0.556667
9	-0.24333	0.156667	0.99
10	-0.24333	0.59	0.99

11	-0.24333	0.99	0.156667
12	-0.24333	0.99	0.59
13	0.09	-0.31	0.923333
14	0.09	0.923333	-0.31
15	0.123333	-0.27667	0.956667
16	0.123333	0.956667	-0.27667
17	0.156667	-0.24333	0.99
18	0.156667	0.99	-0.24333
19	0.523333	-0.31	0.923333
20	0.523333	0.923333	-0.31
21	0.556667	-0.27667	0.956667
22	0.556667	0.956667	-0.27667
23	0.59	-0.24333	0.99
24	0.59	0.99	-0.24333
25	0.923333	-0.31	0.09
26	0.923333	-0.31	0.523333
27	0.923333	0.09	-0.31
28	0.923333	0.523333	-0.31
29	0.956667	-0.27667	0.123333

30	0.956667	-0.27667	0.556667
31	0.956667	0.123333	-0.27667
32	0.956667	0.556667	-0.27667
33	0.99	-0.24333	0.156667

34	0.99	-0.24333	0.59
35	0.99	0.156667	-0.24333
36	0.99	0.59	-0.24333

In fact, we can find 252 systems of three particles which have the same total gravity potential energy.

As you can see, when the distances increase, the work done on systems will decrease.

[The Distances among The Particles in The Space \(2\)](#)

<https://emfps.blogspot.com/2018/04/the-distances-among-particles-in-space-2.html>

You can preview the conjectures (1) and (2) and (3) on below link:

<http://www.emfps.org/2018/04/the-distances-among-particles-in-space.html>

Conjecture (4):

The distances between the point $P_0 (a, a, a)$ and the points mentioned in conjecture (3) are always the same where “ a ” is a member of real number.

Example:

Suppose we have point P_1 with below coordination:

$P_1 (-3, 15, 34)$

According to conjecture (3), the points P2 and P3 will be:

P2 (34, -3, 15)

P3 (15, 34, -3)

The distances among the points P1 and P2 and P3 are equal to 45.32108.

Assume we have point P0 with below coordination:

P0 (56, 56, 56)

The distances between the point P0 (56, 56, 56) and the points P1 and P2 and P3 is the same and

equal to 75.13987.

$$d(P0, P1) = 75.13987$$

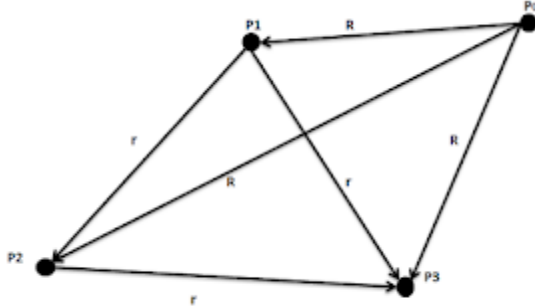
$$d(P0, P2) = 75.13987$$

$$d(P0, P3) = 75.13987$$

The Mapping A System of Four Particle for Given Gravity Potential Energy

Suppose we have four particles P0, P1, P2 and P3 with the distances among them in infinity which have the same masses of "m". If an external force brings all these particles in new location just like

below figure, how can we map the location of these particles for a constant gravity potential energy?



As I told you, suppose that mass of all particles is the same. Therefore, for our analysis, we have three independent variables of “m”, “r” and “R”.

Suppose the constant gravity potential energy is equal $-1.008E-08$ J

$$U(r) = -1.008E-08 \text{ J}$$

As I stated in my previous article, to find the coordination and mapping the particles, we should take two steps:

Step (1):

To find the mass of particles accompanied by the distances of “r” and “R”, we should apply the method mentioned in article of “Solving a Nonlinear Equation with Many Independent Variables” (<http://www.emfps.org/2016/10/can-we-solve-nonlinear-equation-with.html>)

I found 8 answers for three independent variables of the particles:

m	r	R	T_G_P_E
1	0.02	2.6	-1.008E-08
1	0.02	2.7	-1.008E-08
1	0.02	2.8	-1.008E-08
3	0.19	3	-1.008E-08
3	0.21	1.2	-1.008E-08
3	0.23	0.8	-1.008E-08
3	0.24	0.7	-1.008E-08
4	0.36	2.7	-1.008E-08

For example, I choose below three variables from above table:

m	r	R	T_G_P_E
1	0.02	2.8	-1.007949E-08
3	0.21	1.2	-1.007949E-08
3	0.24	0.7	-1.007949E-08

And by using these three set, I start step 2.

Step (2):

Since we should get the coordination of particles by using the distances “r” and “R”, therefore, we have to solve a system of two nonlinear equations as follows:

$$\begin{cases} (x - z)^2 + (y - x)^2 + (z - y)^2 = r^2 \\ (x - a)^2 + (y - a)^2 + (z - a)^2 = R^2 \end{cases}$$

By using the model presented in article of “A Model to Track the Location of a Particle in the Space” (<http://www.emfps.org/2018/02/a-model-to-track-location-of-particle.html>), we can easily find the coordination of the particles.

For instance, if we have:

$$m = 1\text{kg}$$

$$r = 0.02 \text{ m}$$

$$R = 2.8 \text{ m}$$

I found 444 models that some of them are as follows:

Models	x	y	z	a	Distance
1	-0.02	-0.01692	-0.00462	1.602721	2.8
2	-0.02	-0.01179	-0.00359	1.604772	0.02
3	-0.02	-0.00769	-0.00462	1.605798	
4	-0.02	-0.00462	-0.01692	1.602721	
5	-0.02	-0.00462	-0.00769	1.605798	
6	-0.02	-0.00359	-0.01179	1.604772	
7	-0.01897	-0.0159	-0.00359	1.603747	
8	-0.01897	-0.01077	-0.00256	1.605798	
9	-0.01897	-0.00667	-0.00359	1.606823	
10	-0.01897	-0.00359	-0.0159	1.603747	
11	-0.01897	-0.00359	-0.00667	1.606823	
12	-0.01897	-0.00256	-0.01077	1.605798	
13	-0.01795	-0.01487	-0.00256	1.604772	
14	-0.01795	-0.00974	-0.00154	1.606823	
15	-0.01795	-0.00564	-0.00256	1.607849	
16	-0.01795	-0.00256	-0.01487	1.604772	
17	-0.01795	-0.00256	-0.00564	1.607849	
18	-0.01795	-0.00154	-0.00974	1.606823	
19	-0.01692	-0.02	-0.00462	1.602721	
20	-0.01692	-0.01385	-0.00154	1.605798	
21	-0.01692	-0.00872	-0.00051	1.607849	
22	-0.01692	-0.00462	-0.02	1.602721	
23	-0.01692	-0.00462	-0.00154	1.608875	
24	-0.01692	-0.00154	-0.01385	1.605798	
25	-0.01692	-0.00154	-0.00462	1.608875	

For testing of these models, I use model (1) in above table and we can see the results as follows:

	x	y	z	Distance
P1	-0.0200	-0.0169	-0.0046	0.01994
P2	-0.0046	-0.0200	-0.0169	0.01994
P3	-0.0169	-0.0046	-0.0200	0.01994
P0 - P3	1.60272	1.60272	1.60272	2.80000
			P0 - P1	2.80000
			P0 - P2	2.80000

If we have:

$$m = 3\text{kg}$$

$$r = 0.21\text{ m}$$

R = 1.2 m

I found 276 models that some of them are as follows:

Models	x	y	z	a	Distance
1	9.79	9.82231	9.95154	10.5439	1.2
2	9.79	9.91923	9.95154	10.5762	0.21
3	9.79	9.95154	9.82231	10.5439	
4	9.79	9.95154	9.91923	10.5762	
5	9.80077	9.83308	9.96231	10.5547	
6	9.80077	9.96231	9.83308	10.5547	
7	9.81154	9.84385	9.97308	10.5654	
8	9.81154	9.94077	9.97308	10.5978	
9	9.81154	9.97308	9.84385	10.5654	
10	9.81154	9.97308	9.94077	10.5978	
11	9.82231	9.79	9.95154	10.5439	
12	9.82231	9.85462	9.98385	10.5762	
13	9.82231	9.95154	9.79	10.5439	
14	9.82231	9.95154	9.98385	10.6085	
15	9.82231	9.98385	9.85462	10.5762	
16	9.82231	9.98385	9.95154	10.6085	
17	9.83308	9.80077	9.96231	10.5547	
18	9.83308	9.86538	9.99462	10.587	
19	9.83308	9.96231	9.80077	10.5547	
20	9.83308	9.96231	9.99462	10.6193	
21	9.83308	9.99462	9.86538	10.587	
22	9.83308	9.99462	9.96231	10.6193	
23	9.84385	9.81154	9.97308	10.5654	
24	9.84385	9.87615	10.0054	10.5978	
25	9.84385	9.97308	9.81154	10.5654	
26	9.84385	9.97308	10.0054	10.6301	
27	9.84385	10.0054	9.87615	10.5978	
28	9.84385	10.0054	9.97308	10.6301	

For testing of these models, I use model (1) in above table and we can see the results as follows:

	x	y	z	Distance
P1	9.7900	9.8223	9.9515	0.20938
P2	9.9515	9.7900	9.8223	0.20938
P3	9.8223	9.9515	9.7900	0.20938
P0 -P3	10.54391	10.54391	10.54391	1.20000
			P0 -P1	1.20000
			P0 -P2	1.20000

If we have:

$m = 3\text{kg}$

$r = 0.24\text{ m}$

$R = 0.7\text{ m}$

I found 276 models that some of them are as follows:

Models	x	y	z	a	Distance
1	0.751	0.787923	0.935615	1.221042	0.7
2	0.751	0.898692	0.935615	1.257965	0.24
3	0.751	0.935615	0.787923	1.221042	
4	0.751	0.935615	0.898692	1.257965	
5	0.763308	0.800231	0.947923	1.23335	
6	0.763308	0.947923	0.800231	1.23335	
7	0.775615	0.812538	0.960231	1.245657	
8	0.775615	0.923308	0.960231	1.282581	
9	0.775615	0.960231	0.812538	1.245657	
10	0.775615	0.960231	0.923308	1.282581	
11	0.787923	0.751	0.935615	1.221042	
12	0.787923	0.824846	0.972538	1.257965	
13	0.787923	0.935615	0.751	1.221042	
14	0.787923	0.935615	0.972538	1.294888	
15	0.787923	0.972538	0.824846	1.257965	
16	0.787923	0.972538	0.935615	1.294888	
17	0.800231	0.763308	0.947923	1.23335	
18	0.800231	0.837154	0.984846	1.270273	
19	0.800231	0.947923	0.763308	1.23335	
20	0.800231	0.947923	0.984846	1.307196	
21	0.800231	0.984846	0.837154	1.270273	
22	0.800231	0.984846	0.947923	1.307196	
23	0.812538	0.775615	0.960231	1.245657	
24	0.812538	0.849462	0.997154	1.282581	
25	0.812538	0.960231	0.775615	1.245657	

For testing of these models, I use model (1) in above table and we can see the results as follows:

	x	y	z	Distance
P1	0.7510	0.7879	0.9356	0.23929
P2	0.9356	0.7510	0.7879	0.23929
P3	0.7879	0.9356	0.7510	0.23929
P0 -P3	1.22104	1.22104	1.22104	0.70000
			P0 -P1	0.70000
			P0 -P2	0.70000

Conclusion

Suppose you are squeezing a sponge by your hand and assume that the work done by your hand will stay the constant. How can you say that you are controlling the potential energy throughout the sponge when you deform the sponge and change the location of particles of the sponge?



In physics, the people usually use the vector fields, gradient vectors, curl and so on. But the problem is, to encounter with the chaos systems in the nature in which you are not able to find a real function for your subject. In this case, the people usually use the methods of the boundary conditions.

I think that the method mentioned in above article can help us to solve the problems which are defined as the chaos systems.

[The Distances among The Particles in The Space \(3\)](#)

<https://emfps.blogspot.com/2018/04/the-distances-among-particles-in-space-3.html>

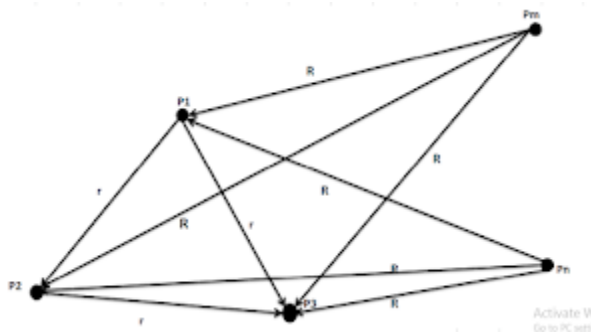
You can preview the conjectures (1), (2), (3) and (4) on below links:

<http://www.emfps.org/2018/04/the-distances-among-particles-in-space.html>

<http://www.emfps.org/2018/04/the-distances-among-particles-in-space-2.html>

Conjecture (5):

By using this theorem or conjecture, I am willing to show you that there is below figure among five points in the space:



Suppose we have point $P_1(x, y, z)$ and an independent variable “ t ” where there is below functions between them:

$$f(x, y, z, t) = \frac{2(x + y + z) + \sqrt{t^2 - 8(x - y - z)^2}}{6}$$

$$g(x, y, z, t) = \frac{2(x + y + z) - \sqrt{t^2 - 8(x - y - z)^2}}{6}$$

Where:

$$t \geq |2\sqrt{2}(x - y - z)| \quad \text{and} \quad x, y, z, t \in R$$

If points P_m and P_n have below coordination:

$P_m (f(x, y, z, t), f(x, y, z, t), f(x, y, z, t))$

$P_n (g(x, y, z, t), g(x, y, z, t), g(x, y, z, t))$

Then, in according to conjecture (3), the distance of points P_m and P_n will be equal with the points P_1, P_2 and P_3 stated in conjecture (3). (Please see conjecture (3) on above links).

It means:

$$d(P_m, P_1) = d(P_m, P_2) = d(P_m, P_3) = d(P_n, P_1) = d(P_n, P_2) = d(P_n, P_3) = R$$

Example (1):

Suppose we have below data:

P1 (-11, 3, 31)

t = 129

The results will be as follows:

	x	y	z	t
P1	-11	3	31	129
P2	31	-11	3	129
P3	3	31	-11	129
<i>f(x,y,z,t)</i>	11.16667			
<i>g(x,y,z,t)</i>	4.16667			
	x	y	z	
Pm	11.16667	11.16667	11.16667	
Pn	4.16667	4.16667	4.16667	
d (Pm,P1)	30.845			
d (Pm,P2)	30.845			
d (Pm,P3)	30.845			
d (Pn,P1)	30.845			
d (Pn,P2)	30.845			
d (Pn,P2)	30.845			

Example (2):

Suppose we have below data:

P1 (-3.57, 9.4, -1.84)

$t = 32.83$

The results will be as follows:

	x	y	z	t
P1	-3.57	9.4	-1.84	32.83
P2	-1.84	-3.57	9.4	32.83
P3	9.4	-1.84	-3.57	32.83
<i>f(x,y,z,t)</i>	2.88272			
<i>g(x,y,z,t)</i>	-0.22272			
	x	y	z	
Pm	2.88272	2.88272	2.88272	
Pn	-0.22272	-0.22272	-0.22272	
d (Pm,P1)	10.3158			
d (Pm,P2)	10.3158			
d (Pm,P3)	10.3158			
d (Pn,P1)	10.3158			
d (Pn,P2)	10.3158			
d (Pn,P2)	10.3158			

The Mapping A System of five Particle for Given Gravity Potential Energy

Suppose we have five particles P_m , P_n , P_1 , P_2 and P_3 with the distances among them in infinity which have the same masses of “ m ”. If an external force brings all these particles in new location just like above figure, how can we map the location of these particles for a constant gravity potential

energy?

The method of analysis is just like the steps stated in previous article

(<http://www.emfps.org/2018/04/the-distances-among-particles-in-space-2.html>)

The only difference is to solve a system of three nonlinear equations instead of a system of two nonlinear equations in previous articles as follows:

$$\begin{cases} (x - z)^2 + (y - x)^2 + (z - y)^2 = r^2 \\ f(x, y, z, t) = a \\ g(x, y, z, t) = b \end{cases}$$

Example:

To simplify above system of equations, I consider “t” as a constant number.

Assume we have:

$$r = 0.24 \text{ m}$$

$$t = 3$$

Thus, we should solve below system of three equations:

$$\begin{cases} (x - z)^2 + (y - x)^2 + (z - y)^2 = r^2 \\ f(x, y, z) = a \\ g(x, y, z) = b \end{cases}$$

I found 190 models that some of them are as follows:

Model	x	y	z	t	$f(x,y,z)$	$g(x,y,z)$
1	0.751	0.78792	0.93562	3	1.024384	0.6253084
2	0.751	0.93562	0.78792	3	1.024384	0.6253084
3	0.76331	0.80023	0.94792	3	1.022793	0.651515
4	0.76331	0.94792	0.80023	3	1.022793	0.651515
5	0.77562	0.81254	0.96023	3	1.019876	0.6790475
6	0.77562	0.96023	0.81254	3	1.019876	0.6790475
7	0.78792	0.751	0.93562	3	1.090407	0.5592851
8	0.78792	0.82485	0.97254	3	1.015238	0.7083002
9	0.78792	0.93562	0.751	3	1.090407	0.5592851
10	0.78792	0.97254	0.82485	3	1.015238	0.7083002
11	0.80023	0.76331	0.94792	3	1.093226	0.5810814
12	0.80023	0.83715	0.98485	3	1.008231	0.7399231
13	0.80023	0.94792	0.76331	3	1.093226	0.5810814
14	0.80023	0.98485	0.83715	3	1.008231	0.7399231
15	0.81254	0.77562	0.96023	3	1.095543	0.6033798
16	0.81254	0.84946	0.99715	3	0.997625	0.7751442
17	0.81254	0.96023	0.77562	3	1.095543	0.6033798
18	0.81254	0.99715	0.84946	3	0.997625	0.7751442
19	0.82485	0.78792	0.97254	3	1.097294	0.6262444
20	0.82485	0.86177	1.00946	3	0.98045	0.8169349
21	0.82485	0.97254	0.78792	3	1.097294	0.6262444
22	0.82485	1.00946	0.86177	3	0.98045	0.8169349
23	0.83715	0.80023	0.98485	3	1.098399	0.6497548
24	0.83715	0.87408	1.02177	3	0.941443	0.8805566
25	0.83715	0.98485	0.80023	3	1.098399	0.6497548
26	0.83715	1.02177	0.87408	3	0.941443	0.8805566

For testing of these models, I use model (1) in above table and we can see the results as follows:

	x	y	z	t
P1	0.751	0.78792	0.93562	3
P2	0.93562	0.751	0.78792	3
P3	0.78792	0.93562	0.751	3
r	0.24			
<i>f(x,y,z)</i>	1.02438			
<i>g(x,y,z)</i>	0.62531			
	x	y	z	
Pm	1.02438	1.02438	1.02438	
Pn	0.62531	0.62531	0.62531	
d (Pm,P1)	0.3722			
d (Pm,P2)	0.3722			
d (Pm,P3)	0.3722			
d (Pn,P1)	0.3722			
d (Pn,P2)	0.3722			
d (Pn,P3)	0.3722			