# A Model to Solve a System of SHM Equations

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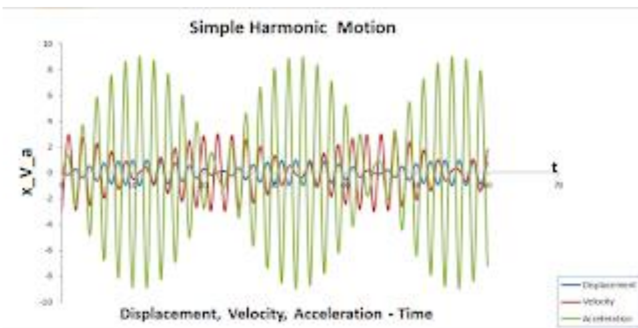
**https://emfps.blogspot.com/2018/08/a-model-to-solve-system-of-shm-equations_23.html**




Following to article of "A Model to Solve a System of Nonlinear Equations by Using Microsoft Excel plus VBA" (http://emfps.blogspot.com/2018/02/a-model-to-track-location-of-particle.html), the purpose of this article is, to present another example about solving a system of nonlinear equations for Simple Harmonic Motion (SHM).

Suppose we have two oscillatory motions which are in two perpendicular directions. We can write a system of equations for them as follows:

$$\begin{cases} x = A_x Cos(\omega_x t + \varphi_x) \\ y = A_y Cos(\omega_y t + \varphi_y) \end{cases}$$

Assume the angular frequency for both SHMs is the same and also there is the difference between initial phases equal 90 degree. In this case, we will have below system of equations:

$$\omega_x = \omega_y \; and \; \varphi_y = \varphi_x + \frac{\pi}{2}$$

$$\begin{cases} x = A_x Cos(\omega t + \varphi_x) \\ y = -A_y Sin(\omega t + \varphi_x) \end{cases}$$

This model is able to solve above system of two equations for given data of “x”, “y”, “Ax” and “Ay”.

There are two methods for finding “ω”, “t” and “ϕ”.

**Method (1):**

The convert of two equations to one equation and solving only one equation for three independent variables in accordance with the method mentioned in article of “Solving a Nonlinear Equation with Many Independent Variables by Using Microsoft Excel plus VBA” (https://emfps.blogspot.com/2016/10/can-we-solve-nonlinear-equation-with.html) as follows:

$$Tan(\omega t + \varphi_x) = -\frac{y.A_x}{x.A_y}$$

Below figure as well as shows you the components of this model:

| | G | H | I | J | K | L | M | N | O | P |
|---|---|---|---|---|---|---|---|---|---|---|
| 4 | | Inputs | | | | Inputs | | | | |
| 5 | | ω | t | ϕ | | x | y | Ax | Ay | |
| 6 | Lower range | -3.14 | 0.10 | -3.14 | | 1 | -7 | 2 | 9 | |
| 7 | Upper range | 3.14 | 1.00 | 3.14 | | error | 0.0001 | | | |
| 8 | | Outptus | | | | | | | | |
| 9 | | ω | t | ϕ | x =AxCos(ωt+ϕ) | | y = -AySin(ωt+ϕ) | | Δx | Δy |
| 10 | Answers 1 | 2.98 | 0.15 | 0.56 | 1.081467251 | | -7.570744933 | | 0.081467 | 0.570745 |
| 11 | Answers 2 | 2.50 | 0.56 | -0.40 | 1.081467251 | | -7.570744933 | | 0.081467 | 0.570745 |
| 12 | Answers 3 | -2.34 | 0.33 | -1.37 | -1.081479611 | | 7.57070918 | | False | False |
| 13 | Answers 4 | 1.85 | 0.93 | -0.72 | 1.081467251 | | -7.570744933 | | 0.081467 | 0.570745 |
| 14 | Answers 5 | 1.53 | 0.28 | 0.56 | 1.081467251 | | -7.570744933 | | 0.081467 | 0.570745 |
| 15 | Answers 6 | 1.37 | 0.38 | -2.66 | -1.081479611 | | 7.57070918 | | False | False |
| 16 | Answers 7 | 0.24 | 0.47 | 0.89 | 1.081467251 | | -7.570744933 | | 0.081467 | 0.570745 |
| 17 | Answers 8 | ω | 0.00 | 0.00 | #VALUE! | | #VALUE! | | #VALUE! | #VALUE! |
| 18 | Answers 9 | ω | 0.00 | 0.00 | #VALUE! | | #VALUE! | | #VALUE! | #VALUE! |
| 19 | Answers 10 | ω | 0.00 | 0.00 | #VALUE! | | #VALUE! | | #VALUE! | #VALUE! |
| 20 | Answers 11 | ω | 0.00 | 0.00 | #VALUE! | | #VALUE! | | #VALUE! | #VALUE! |
| 21 | Answers 12 | ω | 0.00 | 0.00 | #VALUE! | | #VALUE! | | #VALUE! | #VALUE! |
| 22 | Answers 13 | ω | 0.00 | 0.00 | #VALUE! | | #VALUE! | | #VALUE! | #VALUE! |
| 23 | Answers 14 | ω | 0.00 | 0.00 | #VALUE! | | #VALUE! | | #VALUE! | #VALUE! |
| 24 | Answers 15 | ω | 0.00 | 0.00 | #VALUE! | | #VALUE! | | #VALUE! | #VALUE! |

Let me explain you about the components of above model as follows:

1. In right side on cells L6:O6, we have inputs including given data of “x”, “y”, “Ax” and “Ay”.

2. In left side on cells H6:J7, we have other inputs including lower and upper ranges for independent variables of “ω”, “t” and “ϕ” to reach the answers which are the solution for system of two nonlinear equations. Here, there are lower and upper ranges which are changed by click on cell A2 and also this change will again go back by click on cell B2 (Go & Back).

3. On cells H10:J24, we have outputs which are the answers to above system of two nonlinear equations.

4. On cell M7, we have Error which is the difference between equation of outputs (-y.Ax /x.Ay) and equation of inputs (Tan (ωt+ϕ))

5. On cells K10:K24, we have the solution of “x” by replacing the answers.

6. On cells M10:M24, we have the solution of “y” by replacing the answers.

7. On cells O10:O24, we have the difference between item 5 (“x”) and cell L6 which are the errors of our answers.

8. On cells P10:P24, we have the difference between item 6 (“y”) and cell M6 which are the errors of our answers.

**Method (2):**

Using the method stated in article of “A Model to Solve a System of Nonlinear Equations by Using Microsoft Excel plus VBA” ([http://emfps.blogspot.com/2018/02/a-model-to-track-location-of-particle.html](http://emfps.blogspot.com/2018/02/a-model-to-track-location-of-particle.html)

In this method, we directly solve a system of two nonlinear equations.

Below figure as well as shows you the components of this model:

| | G | H | I | J | K | L | M | N | O | P |
|---|---|---|---|---|---|---|---|---|---|---|
| 4 | | Inputs | | | | Inputs | | | | |
| 5 | | ω | t | ϕ | | x | y | Ax | Ay | |
| 6 | Lower range | -5.14 | 0.11 | -5.14 | | 1 | -7 | 2 | 9 | |
| 7 | Upper range | 4.68 | 1.20 | 4.68 | | error | 0.203 | | | |
| 8 | | Outptus | | | | | | | | |
| 9 | | ω | t | ϕ | x =AxCos(ωt+ϕ) | | y = -AySin(ωt+ϕ) | | Δx | Δy |
| 10 | Answers 1 | 4.18 | 0.67 | -1.87 | 1.199397139 | | -7.202013856 | | 0.199397 | 0.202034 |
| 11 | Answers 2 | -4.13 | 0.42 | -3.63 | 1.200267031 | | -7.199098613 | | 0.200267 | 0.199099 |
| 12 | Answers 3 | 3.93 | 0.84 | 3.93 | 1.199653899 | | -7.201167826 | | 0.199654 | 0.201168 |
| 13 | Answers 4 | 3.93 | 0.84 | 3.93 | 1.199653899 | | -7.201167826 | | 0.199654 | 0.201168 |
| 14 | Answers 5 | -3.88 | 0.25 | -4.39 | 1.199419998 | | -7.201956768 | | 0.19942 | 0.201957 |
| 15 | Answers 6 | 3.67 | 0.42 | -0.61 | 1.202258935 | | -7.192364872 | | 0.202259 | 0.192365 |
| 16 | Answers 7 | 3.67 | 0.42 | -0.61 | 1.202258935 | | -7.192364872 | | 0.202259 | 0.192365 |
| 17 | Answers 8 | -3.63 | 0.75 | -2.62 | 1.199894305 | | -7.200356695 | | 0.199894 | 0.200357 |
| 18 | Answers 9 | 3.42 | 0.67 | -1.36 | 1.202186124 | | -7.19261132 | | 0.202186 | 0.192611 |
| 19 | Answers 10 | -3.38 | 0.34 | -4.89 | 1.199819873 | | -7.200607856 | | 0.19982 | 0.200608 |
| 20 | Answers 11 | -3.38 | 0.34 | -4.89 | 1.199819873 | | -7.200607856 | | 0.19982 | 0.200608 |
| 21 | Answers 12 | 3.17 | 0.17 | 0.40 | 1.202641383 | | -7.191069982 | | 0.202641 | 0.19107 |
| 22 | Answers 13 | -3.13 | 1.12 | -1.87 | 1.201694294 | | -7.194275446 | | 0.201694 | 0.194275 |
| 23 | Answers 14 | -2.87 | 0.25 | -4.64 | 1.199833407 | | -7.200562192 | | 0.199833 | 0.200562 |
| 24 | Answers 15 | -2.87 | 0.25 | -4.64 | 1.199833407 | | -7.200562192 | | 0.199833 | 0.200562 |

The explanations of the components are the same with method (1) except the error.