Analysis and Design Open Oscillatory Systems with Forced Harmonic Motion

By: Gholamreza Soleimani

https://emfps.blogspot.com/2014/02/analysis-and-design-open-oscillatory.html

Consider a child, who is playing with a swing. During the period of the time, he learns to apply the optimum force to the swing in order to minimize efforts and maximize the amplitude of the swing. How? The answer is that driving force should be applied periodically and should be timed to coincide closely with the natural motion of the swing. In other words, a driven oscillator responds most strongly when driven by a periodically varying force, the frequency of which is closely matched to the frequency with which the system would freely oscillate if left to it. This frequency is called the natural frequency of the oscillator.

The purpose of this article is to utilize some methodologies such as sensitivity analysis and Monte Carlo simulation model to analyse and design open systems which have the damped harmonic motion and are also forced by external oscillatory forces. A case of *"Is There Any Mechanical Oscillatory System Where Maximum Velocity of Resonance Will Increase More Than Speed of the Light?"* has been analysed by using of the methodology stated in article of *"EMFPS: How Can We Get the Power Set of a Set by Using of Excel?"* posted on link: http://emfps.blogspot.com/2012/08/emfps-how-can-we-get-power-set-of-set.html.

Introduction

There are three types of oscillatory motions as follows:

1. Mechanical waves: These involve motions that are governed by Newton's laws and can exist only within a material medium such as air, water, rock, etc. Common examples are: sound waves, seismic waves, etc.

2. Matter (or material) waves: All microscopic particles such as electrons, protons, neutrons, atoms etc. have a wave associated with them governed by *Schrödinger's equation*.

3. Electromagnetic waves: These waves involve propagating disturbances in the electric and magnetic field governed by Maxwell's equations. They do not require a material medium in which to propagate but they travel through vacuum. Common examples are: radio waves of all types, visible, infra-red, and ultra-violet light, x-rays, and gamma rays. All electromagnetic waves propagate in vacuum with the same speed of the light(c = 300,000 km/s).

First of all, I am willing to start the analysis and design of a mechanical system which is harmonically moving and it has been referred to Mechanical waves (Item 1). Before that, let me tell you a summary of damped and forced SHM.

Damped Harmonic Motion:

We know that a SHM can infinitely continue its motion, if there is not any friction force. In this case, a mass connected to a spring will have oscillatory motion forever. But the amplitude of SHM usually decreases and is closed to zero due to friction force. We say that is a Damped Harmonic Motion (DHM). The damped force depends on the velocity of the particle and it can be calculated from formula: - b(dx/dt) where "b" is a positive constant number. The equation of the motion is obtained by using of Newton's laws(F = ma) as follows:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$x = Ae^{-\frac{bt}{2m}}cos(\omega' t + \varphi)$$

$$\omega' = 2\pi\nu' = \sqrt{\frac{k}{m} - (\frac{b}{2m})^2}$$

Reference: K. R. Symon, Mechanics. Third edition, Addison – Wesley Publishing Company, 1971, Section 2.9.

Forced Harmonic Motion (FHM):

But if an external oscillatory force is affecting on an open system with DHM, we can analyze the equation of motion in accordance with below formula:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_m cos\omega''t$$

$$x = \frac{F_m}{G} sin\,(\omega''t - \varphi)$$

$$G = \sqrt{m^2\left(\omega''^2 - \omega^2\right)^2 + b^2\omega''^2}$$

$$\varphi = cos^{-1}\frac{b\omega''}{G}$$

Reference: K. R. Symon, Mechanics. Third edition, Addison – Wesley Publishing Company, 1971, Section 10.2.

In this case, when the frequency of external force reaches to natural frequency of our system, we will have the resonance.

Regarding to above equations, we can see that the most important parameters for analysis and designing of an open system are as follows:

Fm = External force (N)

k = Restoring constant of system (N/m)

m = mass of system (kg)

b = Damped force constant of system (kg/s)

ω" = Angular velocity of external force (rad/s)

Methodologies

I used from three methods in which each one is assigned to one type of the oscillatory motions as follows:

- For mechanical waves, I consider to utilize the method mentioned in article of *"EMFPS: How Can We Get the Power Set of a Set by Using of Excel?"* posted on link: http://emfps.blogspot.com/2012/08/emfps-how-can-we-get-power-set-of-set.html.As an example, I will analyze a case by using of this method where the result will be the options for designing.

- For matter (or material) waves, I will use fromMonte Carlo simulation method stated in my previous articles such as *"Application of Pascal's Triangular plus Monte Carlo Analysis to Find the Least Squares Fitting for a Limited Area"* posted on link: http://emfps.blogspot.com/2012/05/application-of-pascals-triangular-plus_23.html.As an example, I will examine the oscillatory motion of a free neutron to find out its coordination in related with the time.

-For electromagnetic waves, I will utilize from Sensitivity Analysis and as an example, I will analyze a case of energy carried by Gamma ray.

1. A Case of Mechanical Waves

Case*: Is There Any Mechanical Oscillatory System Where Maximum Velocity of Resonance Will Increase More Than Speed of the Light?"*

Assume we are designing an open system under force harmonic motion. What are the parameters of designing? According to above mentioned, they are as follows:

Fm = External force (N)

k = Restoring constant of system (N/m)

m = mass of system (kg)

b = Damped force constant of system (kg/s)

ω" = Angular velocity of external force (rad/s)

We are willing to know if there is any mechanical system with FHM in which maximum velocity of this system will go up more than 3E+8 m/s. What is the range for parameters of designing?

I used from the method stated in article of *"EMFPS: How Can We Get the Power Set of a Set by Using of Excel?"* posted on link: http://emfps.blogspot.com/2012/08/emfps-how-can-we-get-power-set-of-set.html.

I would like to remind you that we applied VB code written by Myrna Larson where the method of designing is step by step as follows:

- I know that the velocity of our system is the function of the above parameters (independent variables): V = *f* (Fm, k, m, b, ω'') and we need to have Vm> 3E+8 m/s

- I consider a random domain for all five parameters for instance: 0.1 < (Fm, k, m, b, ω'')< 1

- I start my calculation by using of Myrna Larson's VB code and excel spreadsheet program. I have to analyse only 30240 column for calculations simultaneously (=Permutt (10, 5)) because my PC has not necessary instruments to analyse big data.

- I change the domain for all five parameters: 0.001 <(Fm, k, m, b, ω'')< 100

- I continue to change the domain where I reach: 0.000001 ≤ (Fm, k, m, b, ω'') ≤ 1000

In this domain, I found 17 types of the parameters where maximum velocity of our system is equal to 1E+9 m/s > c = 3E+8 m/s. It means that we can have 17 types of design for our system to reach maximum velocity more than speed of the light. All parameters for designing have been arranged in below Table:

Fm	k	m	b	ω''	A (m)	ω''/ω	Vm(m/s)
1000	0.00001	0.001	0.000001	0.1	1E+10	1	1E+09
1000	0.00001	0.1	0.000001	0.01	1E+11	1	1E+09
1000	0.00001	10	0.000001	0.001	1E+12	1	1E+09
1000	0.0001	0.01	0.000001	0.1	1E+10	1	1E+09
1000	0.0001	1	0.000001	0.01	1E+11	1	1E+09
1000	0.0001	100	0.000001	0.001	1E+12	1	1E+09
1000	0.001	0.00001	0.000001	10	1E+08	1	1E+09
1000	0.001	10	0.000001	0.01	1E+11	1	1E+09
1000	0.01	0.0001	0.000001	10	1E+08	1	1E+09
1000	0.01	1	0.000001	0.1	1E+10	1	1E+09
1000	0.1	0.00001	0.000001	100	1.E+07	1	1E+09
1000	0.1	0.001	0.000001	10	1E+08	1	1E+09
1000	1	0.0001	0.000001	100	1.E+07	1	1E+09
1000	1	0.01	0.000001	10	1.E+08	1	1E+09
1000	1	100	0.000001	0.1	1.E+10	1	1E+09
1000	10	0.001	0.000001	100	1.E+07	1	1E+09
1000	100	1	0.000001	10	1E+08	1	1E+09

As we can see, the most crucial thing is that our system will reach to maximum velocity more than speed of the light, if external oscillatory force goes up more than 1KN and damped force constant decrease less than 1E-6 kg/s. In fact, the boundary conditions are:

$Fm \geq 1KN$ and $b \leq 1E\text{-}6$ kg/s

2. A Case of Matter (or material) waves

Case: *How Can We Find the Coordination of Free Neutrons in the Space of Entropy?*

The neutron is electrically neutral as its name implies. Because the neutron has no charge, it was difficult to detect with early experimental apparatus and techniques. Today, neutrons are easily detected with devices such as plastic scintillators. Neutrons are elementary particles with mass m_N= 1.67 × 10−27 kg.

Free neutrons are unstable. They undergo beta-decay where its half-life is approximately between 614 to 885.7 ± 0.8 s. Neutrons emitted in nuclear reactions can be slowed down by collisions with matter. They are referred to as thermal neutrons after they come into thermal equilibrium with the environment. The average kinetic energy of a thermal neutron is approximately 0.04 eV. This moderated (thermal) neutrons move about 8 times the speed of sound. Typical wavelength *(λ)*values for thermal neutrons(also called *on-relativistic neutrons cold*) are between 0.1 and 1 nm. Their properties are described in the framework *of material* *wave mechanics*. Therefore, we can easily calculate *de Broglie wavelength* of these neutrons. But can de Broglie wavelength help us to solve this case? How?

As I stated, the analysis of an oscillatory neutron can be done by *Schrödinger's equation*. The general figure of this equation is as follows:

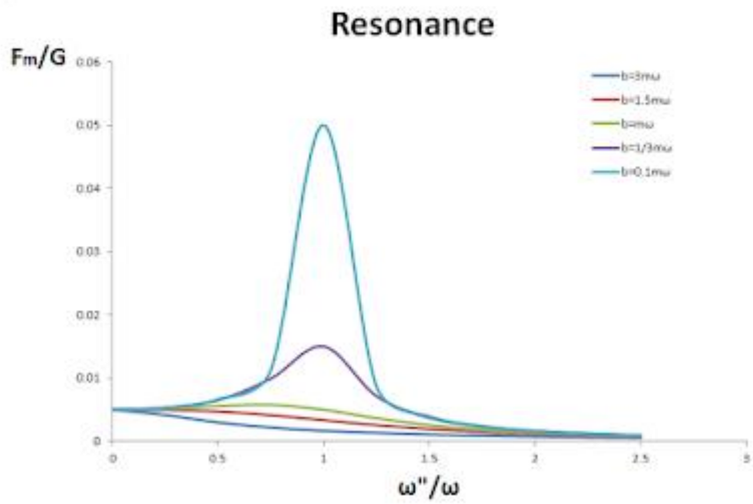
$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+U\psi=E\psi$$

To solve above equation for boundary conditions, we need to apply a strong method. *Can Monte Carlo Simulation method help us to analyse this case?*

For using of Monte Carlo simulation model, I firstly choose the probability distribution inferred from *Binomial* and *Bayesian* method to obtain a framework referred to entropy of these neutrons…

[Analysis and Design Open Oscillatory Systems with Forced Harmonic Motion (2)](https://emfps.blogspot.com/2018/09/analysis-and-design-open-oscillatory.html)

https://emfps.blogspot.com/2018/09/analysis-and-design-open-oscillatory.html



Following to articles of "Analysis and Design Open Oscillatory Systems with Forced Harmonic Motion (1)" (https://emfps.blogspot.com/2014/02/analysis-and-design-open-oscillatory.html) and "A Model to Solve a System of SHM Equations" (https://emfps.blogspot.com/2018/08/a-model-to-solve-system-of-shm-equations_23.html), the purpose of this article is, to find out the characteristics of unknown open oscillatory system which is under forced harmonic motion in the moment of close to resonance. In this case, three models have been designed. One model presents the results for two independent variables "m", "b" and another model gives us the results for three independent variables "m", "b" and "ω"/ω" and the third model shows us the results for higher than a specific velocity.

Statement of the problem

In many situations of oscillatory motions, we have to control the point of the resonance to prevent a huge collapse of system. On the other hand, sometimes we need to know the width of the resonance curve and the characteristics of the system such as natural frequency to increase the velocity of the system by using a forced harmonic motion. In fact, the question is: What is the amount of the force and angular frequency for a forced harmonic motion where the amplitude and velocity of an oscillatory system will significantly increase? These two models are able to answer to this question.

Model (1): The results for two independent variables "mass" and "linear damping constant"

Please be informed that here we encounter with seven independent variables and we have to process all these variables simultaneously for solving the nonlinear equations. For first try, I breakdown the problem and start my analysis by using the most important independent variables which are "m" and "b" and also utilizing the method mentioned in article of "A Model to Solve a System of SHM Equations" (https://emfps.blogspot.com/2018/08/a-model-to-solve-system-of-shm-equations_23.html) accompanied by some tricks in Microsoft excel. I think this is the easiest way. This model says us what are maximum displacement and velocity for a defined range of "m" and "b" close to the point of the resonance.

Below figure as well as shows you the components of this model:

	G	H	I	J	K	L	M
1							
2							
3							
4		Inputs			Inputs		
5		m	b		ω"/ω	Fm	t
6	Lower range	0.05	0.00730		1	400	7.853982
7	Upper range	5.50	0.05500				
8		Outptus		Unit			
9		|Xmax|	8644.1093	m			
10		|Vmax|	7287.3707	m/sec			
11		m	0.0460	kg			
12		b	0.0461	kg/sec			
13		ω	1.0029	rad/sec			
14		k	0.0463	N/m			
15		E	1728388.4	J			
16		A	8646.28	m			
17		Av	8670.93	m/sec			

The components of above model are as follows:

1. In right side on cells K6:M6, we have inputs including given data of "ω″/ω", "Fm", and "t".

2. In left side on cells H6:I7, we have other inputs including lower and upper ranges for independent variables of "m" and "b" to reach the answers which are the solution for driven oscillatory system close to the point of the resonance. Here, there are lower and upper ranges which are changed by click on cell A2 and also this change will again go back by click on cell B2 (Go & Back).

3. On cells I9:I17, we have outputs which are maximum displacement and velocity for driven oscillatory system close to the point of the resonance.

Note: On I9 and I10, we have maximum displacement and velocity for a defined range of "m" and "b". Please do not consider them as the amplitude of harmonic motion. On cell I16 and I17, we have the amplitude of displacement (A) and the amplitude of velocity (Av).

Model (2): The results for three independent variables "mass", "linear damping constant" and "ω″/ω"

In this model, there are the defined ranges for "m", "b" and "ω″/ω" which are our three independent variables and by changing the forced harmonic motion (Fm) and time (t), we are willing to know what are the characteristics of a unknown oscillatory system that give us the maximum displacement and velocity close to the point of the resonance.

Below figure as well as shows you the components of this model:

	G	H	I	J	K	L	M
1							
2							
3							
4		Inputs				Inputs	
5		m	b	ω''/ω		Fm	t
6	Lower range	0.02600	0.00410	0.75		5	1.570796
7	Upper range	3.10000	0.03500	1.25			
8		Outptus		Unit			
9		|Xmax|	194.6155	m			
10		|Vmax|	129.5111	m/sec			
11		m	0.0260	kg			
12		b	0.0263	kg/sec			
13		ω''/ω	0.9423	rad/sec			
14		k	0.0266	N/m			
15		E	503.2166	J			
16		A	216.71	m			
17		Av	190.21	m/sec			

The components of above model are as follows:

1. In right side on cells L6:M6, we have inputs including given data of “Fm”, and “t”.

2. In left side on cells H6:J7, we have other inputs including lower and upper ranges for independent variables of “m”, “b” and “ω″/ω” to reach the answers which are the solution for driven oscillatory system close to the point of the resonance. Here, there are lower and upper ranges which are changed by click on cell A2 and also this change will again go back by click on cell B2 (Go & Back).

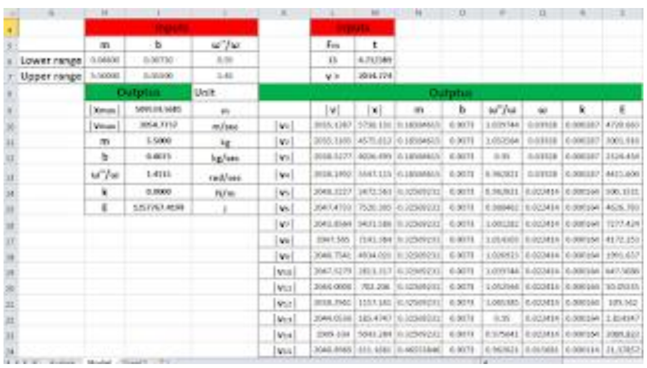
3. On cells J9:J17, we have outputs which are maximum displacement and velocity for driven oscillatory system close to the point of the resonance.

Note: On J9 and J10, we have maximum displacement and velocity for a defined range of “m”, “b” and “ω″/ω”. Please do not consider them as the amplitude of harmonic motion. On cell J16 and J17, we have the amplitude of displacement (A) and the amplitude of velocity (Av).

Model (3): The results of three independent variables "mass", "linear damping constant" and "ω″/ω" for higher than a specific velocity

In this model, we want to design an unknown oscillatory system for a specific velocity. We can have all characteristics of the system for focus, lower and higher than a specific velocity. Here, I consider it for a higher than specific velocity. Since there are many answers, I have only fixed 15 answers for this model.

Below figure as well as shows you the components of this model:



As you can see, all components of this model are the similar to model (2). Only I have added15 absolute value for velocities more than a specific velocity and also on cell M7, we have specific amount of velocity.

Data Analysis and Conclusion

One of the most important applications of above models is to create the value from data analysis. Please see below results:

Fm	t	m	b	ω	ω"	G	ϕ	x	v
5	0	0.05	0.0073	0.1587	0.1528	0.00112	0.07565	-337.822	681.019
5	0	0.19	0.0073	0.03928	0.03782	0.00028	0.07565	-1364.85	681.019
5	0	0.33	0.0073	0.02241	0.02158	0.00016	0.07565	-2391.87	681.019
5	0	0.47	0.0073	0.01568	0.0151	0.00011	0.07565	-3418.89	681.019
5	0	0.61	0.0073	0.01206	0.01161	8.5E-05	0.07565	-4445.92	681.019
5	0	0.75	0.0073	0.0098	0.00943	6.9E-05	0.07565	-5472.94	681.019
5	0	0.89	0.0073	0.00825	0.00794	5.8E-05	0.07565	-6499.96	681.019
5	0	1.02	0.0073	0.00712	0.00686	5E-05	0.07565	-7526.99	681.019
5	0	1.16	0.0073	0.00627	0.00603	4.4E-05	0.07565	-8554.01	681.019
5	0	1.30	0.0073	0.0056	0.00539	3.9E-05	0.07565	-9581.03	681.019
5	0	1.44	0.0073	0.00505	0.00487	3.6E-05	0.07565	-10608.1	681.019
5	0	1.58	0.0073	0.00461	0.00444	3.2E-05	0.07565	-11635.1	681.019
5	0	1.72	0.0073	0.00423	0.00408	3E-05	0.07565	-12662.1	681.019
5	0	1.86	0.0073	0.00392	0.00377	2.8E-05	0.07565	-13689.1	681.019
5	0	2.00	0.0073	0.00364	0.00351	2.6E-05	0.07565	-14716.1	681.019
5	0	2.14	0.0073	0.00341	0.00328	2.4E-05	0.07565	-15743.2	681.019
5	0	2.28	0.0073	0.0032	0.00308	2.3E-05	0.07565	-16770.2	681.019
5	0	2.42	0.0073	0.00301	0.0029	2.1E-05	0.07565	-17797.2	681.019
5	0	2.56	0.0073	0.00285	0.00274	2E-05	0.07565	-18824.2	681.019
5	0	2.70	0.0073	0.0027	0.0026	1.9E-05	0.07565	-19851.3	681.019
5	0	2.84	0.0073	0.00257	0.00247	1.8E-05	0.07565	-20878.3	681.019
5	0	2.98	0.0073	0.00245	0.00236	1.7E-05	0.07565	-21905.3	681.019
5	0	3.12	0.0073	0.00234	0.00225	1.6E-05	0.07565	-22932.3	681.019
5	0	3.26	0.0073	0.00224	0.00215	1.6E-05	0.07565	-23959.4	681.019
5	0	3.40	0.0073	0.00215	0.00207	1.5E-05	0.07565	-24986.4	681.019
5	0	3.54	0.0073	0.00206	0.00198	1.5E-05	0.07565	-26013.4	681.019
5	0	3.68	0.0073	0.00198	0.00191	1.4E-05	0.07565	-27040.4	681.019
5	0	3.82	0.0073	0.00191	0.00184	1.3E-05	0.07565	-28067.5	681.019
5	0	3.96	0.0073	0.00184	0.00177	1.3E-05	0.07565	-29094.5	681.019
5	0	4.10	0.0073	0.00178	0.00171	1.3E-05	0.07565	-30121.5	681.019
5	0	4.24	0.0073	0.00172	0.00166	1.2E-05	0.07565	-31148.5	681.019
5	0	4.38	0.0073	0.00167	0.0016	1.2E-05	0.07565	-32175.5	681.019

Above table shows us that there are many answers for a constant velocity. But if you carefully focus on the results, you will find out an interesting property about driven oscillations and resonance.

What is this property?

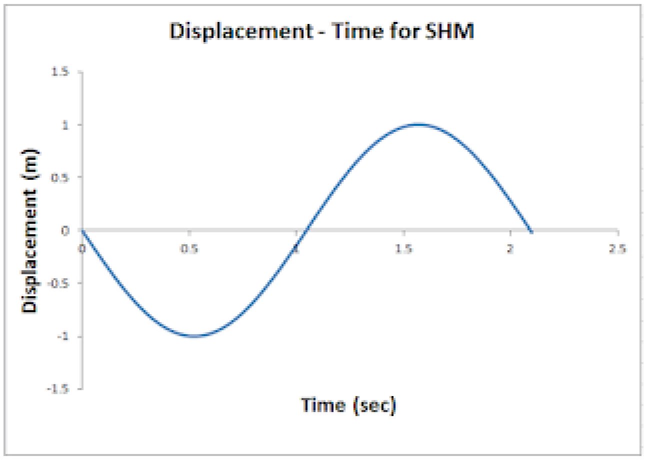
If an oscillatory system has very low angular frequency (ω) and linear damping constant (b) in the conditions close to the point of the resonance and in zero time (t =0), surprisingly by increasing the mass, the displacement (x) will go up and by decreasing the mass, the displacement will go down. In this situation, the amplitude for any changes on mass will stay the constant forever.

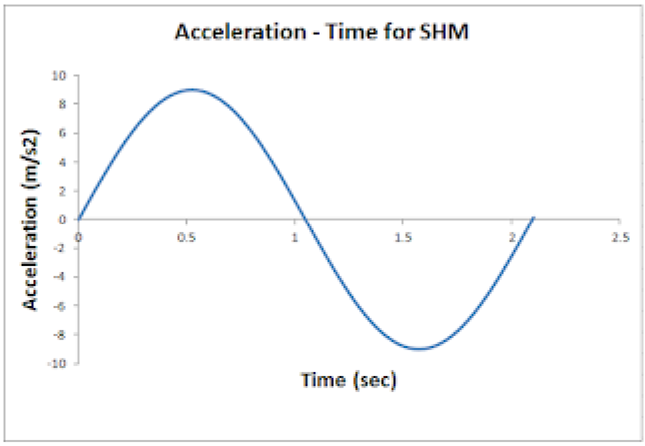
Analysis and Design Open Oscillatory Systems with Forced Harmonic Motion (3)

https://emfps.blogspot.com/2018/09/analysis-and-design-open-oscillatory_25.html

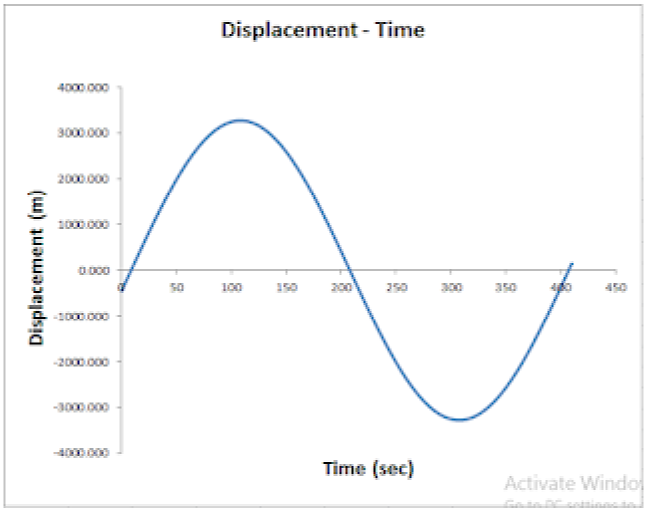
Following the article “Analysis and Design Open Oscillatory Systems with Forced Harmonic Motion (2)” (https://emfps.blogspot.com/2018/09/analysis-and-design-open-oscillatory.html), the purpose of this article is to present a model for finding the characteristics of an oscillatory system with forced harmonic motion where the acceleration will be equal zero. In this case, there is an interesting point that is related to an important difference between SHM and an oscillatory system with forced harmonic motion.

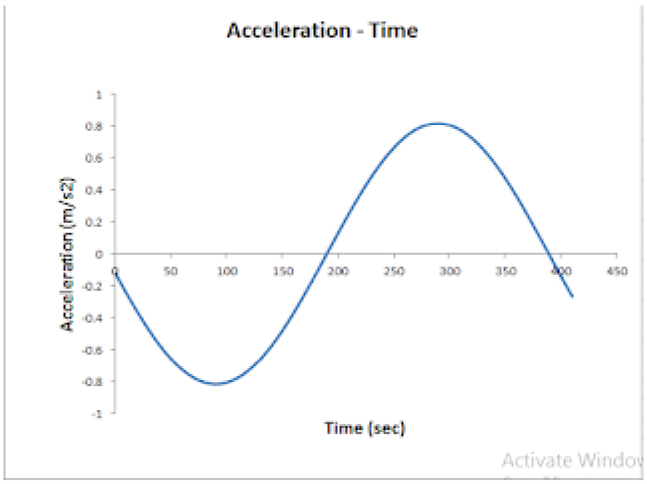
If an object is oscillating under simple harmonic motion, its linear velocity will be zero at the highest and lowest points where we have maximum displacement which is named the amplitude (A).On the other hand; at the maximum level of the displacement (x = A), the acceleration has also its maximum magnitude while at the middle (x = 0), acceleration is zero due to stop at those points in order to change direction while velocity gains its maximum magnitude at the equilibrium point (x = 0). At the extreme ends (x = A), when we have the maximum force and kinetic energy, the acceleration has its maximum magnitude. Therefore, the maximum of acceleration magnitude in simple harmonic motion occurs at maximum displacement (A) and acceleration at the middle is zero when we have the displacement equal to zero just like the below diagrams:





But there is a different story about the oscillatory systems with forced harmonic motion. In this case, at the some points where the displacement is not zero (x = a), we have the acceleration equal to zero. Please see below diagrams:





Now, the question is: What are the characteristics of an oscillatory system with forced harmonic motion where we have the acceleration equal to zero at the point of x = a?

Below model is able to answer above question:

	G	H	I	J	K	L	M
1							
2							
3							
4		Inputs				Inputs	
5		m	b	ω"/ω		Fm	t
6	Lower range	0.02600	0.00410	0.75		1	190
7	Upper range	3.10000	0.03500	1.22			
8		Outptus		Unit			
9		a	4.22E-07	m/s2			
10		m	1.12549	kg			
11		b	0.01915	kg/sec			
12		ω"/ω	0.93077				
13		ω	0.01696	rad/sec			
14		ω"	0.01578	rad/sec			
15		x	921.39	m			
16		A	3274.11846	m			
17		k	0.000324811	N/m			
18		E	1740.962695	J			

The components of above model are as follows:

1. In right side on cells L6:M6, we have inputs including given data of “Fm”, and “t”.

2. In left side on cells H6:J7, we have other inputs including lower and upper ranges for independent variables of “m”, “b” and “ω″/ω” to reach the answers for the acceleration equal to zero which are the characteristics of driven oscillatory system. Here, there are lower and upper ranges which are changed by click on cell A2 and also this change will again go back by click on cell B2 (Go & Back).

3. On cells I9:I18, we have outputs which are the characteristics for driven oscillatory system responding to the acceleration equal to zero.