The Change Depends on the Direction of the Motion: Generating All Directions in 3D Space

By: Gholamreza Soleimani

https://emfps.blogspot.com/2017/05/the-change-depends-on-direction-of.html

In physics science and engineering especially fluid dynamics and electromagnetism fields, we usually need to investigate the changes of a function in different directions. In this case, the best way for analyzing and designing is to have different directions all together in our hand in which we will be able to compare all results to reach the new theorems and the new physical phenomena. The purpose of this article is to make a spreadsheet on excel file by using a new method where you will have all unit vectors in 3D space in different directions.

Example:

Suppose you have a curve in 3D space as follows and the point P (x, y, z) wants to move from P0 (-1, 3, 2) on this curve toward all directions in amount of 4 unit (Δs = 4 unit). The question is:

What is the maximum change of below function when point P (x, y, z) moves in all directions? Which direction will the maximum change of the function occur?

Solution:

$f(x,y,z) = x^2 + y^2 - 2z^2$

$\nabla f = -2i + 6j - 8k$

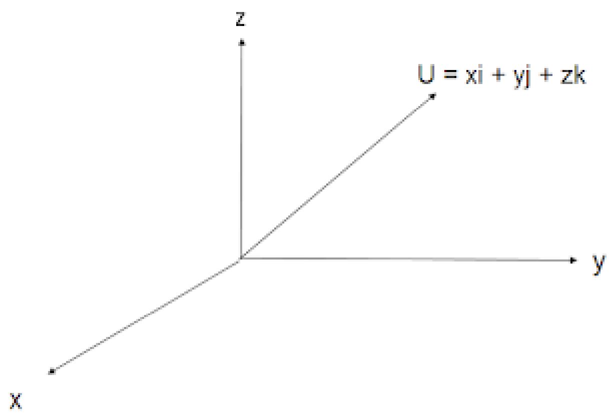
$\Delta f \cong \nabla f . U_i . \Delta s$

$1 \leq i \leq 64442 , i \in N$

The answer is:

$\Delta f_{max} = 40.79105\ \textit{unit, in Direction}\ U = -0.19025i + 0.585529j - 0.78801k$

Unit Vectors in 2D and 3D



You as well as know, we can easily get all unit vectors in 2D surface by using bellow formula:

$U = i \cos\theta + j \sin\theta$

On the other hand, we can obtain the direction of a vector in 2D and 3D, by using below formula:

Direction of $A = A / |A|$

But, how can we prove above formula?

A vector in 3D space can be modelled as the radius of a sphere in which we will have below function:

w = r (x, y, z) = (x^2 + y^2 + z^2) ^0.5

The gradient vector in any point is:

$$\nabla r = \frac{x}{\sqrt{x^2 + y^2 + z^2}} i + \frac{y}{\sqrt{x^2 + y^2 + z^2}} j + \frac{z}{\sqrt{x^2 + y^2 + z^2}} k$$

As you can see, the gradient vector proves above formula (formula:

Direction of A = A / |A|) where vector A = xi + yj + zk because we have:

$$\left| \nabla r \right| = 1$$

Generating all directions in 3D space

Consider a particle starts its circular motion on surface XY and simultaneously has a circular motion on surface XZ perpendicular to surface XY. In this case, this particle will produce a sphere where its circular angle on surface XY is "θ" and its circular angle on surface XZ is "β".

According to above circular angles and radius of sphere (r), we can calculate coordination of point P (x, y, z) on sphere by using below equations:

x = r * cos β * cos θ

y = r * cos β * sin θ

z = r * sin β

Example:

Solve equation: $x^2 + y^2 + z^2 = 225$

If $\theta = 25$ degree and $\beta = 56$ degree

Answer:

We have: $r = (225)^{0.5} = 15$

$x = 15 * \cos 56 * \cos 25 = 7.602013$

$y = 15 * \cos 56 * \sin 25 = 3.544877$

$z = 15 * \sin 56 = 12.43556$

$(7.602013^2) + (3.544877^2) + (12.43556^2) = 225$

In fact, we have below vector:

$V = xi + yj + zk$

$V = (r * \cos\beta * \cos\theta)i + (r * \cos\beta * \sin\theta)j + (r * \sin\beta)k$

For r = 1, we have below unit vector in 3D space:

$U = (\cos\beta * \cos\theta)i + (\cos\beta * \sin\theta)j + (\sin\beta)k$

It is clear; the range of changes for “θ” and “β” is between 0 and 360 degree.

Therefore, for making a spreadsheet included all directions, we should go below steps:

- Choose Δθ and Δβ between 0 and 360 degree. For instance, I considered Δθ = Δβ = 1

- By using the method stated in article of "Can We Solve a Nonlinear Equation with Many Variables?" posted on link: http://emfps.blogspot.co.uk/2016/10/can-we-solve-nonlinear-equation-with.html, we should find all combinations of "θ" and "β" for Δθ and Δβ between 0 and 360 degree. For example, when I considered Δθ = Δβ = 1, I will have 130321 combinations on my excel spreadsheet (361^2 = 130321).

If you choose Δθ = Δβ = 0.5, you will have 519841 combinations (directions) on your excel spreadsheet (721^2 = 519841).

Anyway, I think that Δθ = Δβ = 1 is enough.

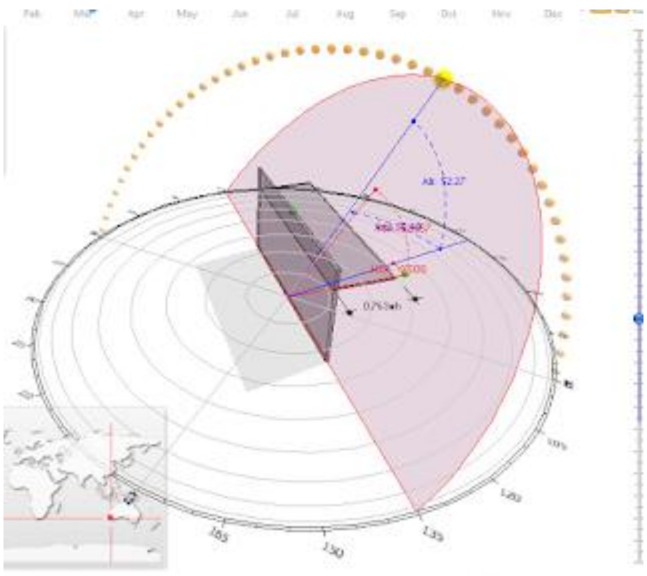
- Using from above equations for r = 1 and each set of combinations. In this case, you have 130321 rows that it show you all directions which you need to your analysis.

In the next articles, I will show you how we can utilize this spreadsheet as a template to investigate the changes of some physical functions.

The Change Depends on the Direction of the Motion: The Angles in Shadow

https://emfps.blogspot.com/2017/06/the-change-depends-on-direction-of.html

In the reference with my article of "The Change Depends on the Direction of the Motion: Generating All Directions in 3D Space" posted on link: https://emfps.blogspot.com/2017/05/the-change-depends-on-direction-of.html , before going to any analysis in physics and engineering subjects, you have to calibrate and filter your template by using the method mentioned in this article. In fact, I had to make the necessary amendments to above article because maybe you will take some mistakes in your analysis when you utilize the template stated in my previous article (Generating All Directions in 3D Space).



What is the case?

The case is to be produced many angles which show the same direction. I name these angles:

“The Angles in Shadow”

What is the meaning “The angles in Shadow”?

Let me tell you an example to illustrate this concept.

Assume you are fixing your telescope on four points in the space. First you turn your telescope on Horizontal angle 104 degree (θ = 104) and Vertical angle 253 (β = 253). Then you turn it on Horizontal angle 284 degree (θ = 284) and Vertical angle 287 (β = 287). Then you turn it on Horizontal angle 104 degree (θ = 104) and Vertical angle 287 (β = 287) and finally you turn it on Horizontal angle 284 degree (θ = 284) and Vertical angle 253 (β = 253). If you use the equations stated in my previous article, you can calculate all directions as follows:

State No.	θ	β	Direction	i	j	k
1	104	253	U1	0.070731	-0.28369	-0.9563
2	284	287	U2	0.070731	-0.28369	-0.9563
3	104	287	U3	-0.07073	0.283687	-0.9563
4	284	253	U4	-0.07073	0.283687	-0.9563

As you can see, you are really looking at two points instead of four points. In fact, state number 1 and 2 are in the same direction and state number 3 and 4 are in the same direction.

How can we find the angles in shadow?

Here I am willing to introduce to you two methods. The method (1), which uses some trigonometric equations while method (2) follows the same method mentioned in article of "Can We Solve a Nonlinear Equation with Many Variables?" posted on link: http://emfps.blogspot.co.uk/2016/10/can-we-solve-nonlinear-equation-with.html

At the first, we should bear in mind that below conditions should be established for both methods:

Condition (1): For Vertical angles,

$\sin\beta = \sin\beta'$ and $\cos\beta = -\cos\beta'$

Condition (2): For Horizontal angles,

$\sin\theta = -\sin\theta'$ and $\cos\theta = -\cos\theta'$

Method (1):

We as well as know Trigonometric reduction formulas which are as follows:

Sin (90 – α) = Cos α

Cos (90 – α) = Sin α

Sin (90 + α) = Cos α

Cos (90 + α) = - Sin α

To obtain condition (1), I use below tricks:

Sin (90 – α) = Cos α = Sin β

Cos (90 – α) = Sin α = Cos β

Sin (90 + α) = Cos α = Sin β'

Cos (90 + α) = - Sin α = - Cos β'

And so, we have below formulas in Trigonometric:

Cos (180 + α) = - Cos α

Sin (180 + α) = - Sin α

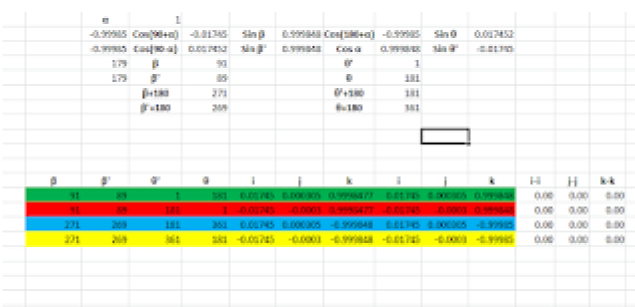
To get condition (2), I also use below formulas:

Cos (180 + α) = - Cos α = - Cos ϑ'

Sin (180 + α) = - Sin α = - Sin ϑ'

ϑ = α and ϑ' = 180 + α

According to above relationships, I can write a simple algorithm to generate all direction including symmetry direction and others as follows:



α	1							
-0.99985	Cos(90+α)	-0.01745	Sin β	0.999848	Cos(180+α)	-0.99985	Sin ϑ	0.017452
-0.99985	Cos(90-α)	0.017452	Sin β'	0.999848	Cos α	0.999848	Sin ϑ'	-0.01745
179	β	91			ϑ'	1		
179	β'	89			ϑ	181		
	β+180	271			ϑ'+180	181		
	β'+180	269			ϑ+180	361		

β	β'	ϑ'	ϑ	i	j	k	i	j	k	i-i	j-j	k-k
91	89	1	181	0.01745	0.000305	0.9998477	0.01745	0.000305	0.999848	0.00	0.00	0.00
91	89	181	1	-0.01745	-0.0003	0.9998477	-0.01745	-0.0003	0.999848	0.00	0.00	0.00
271	269	181	361	0.01745	0.000305	-0.999848	0.01745	0.000305	-0.99985	0.00	0.00	0.00
271	269	361	181	-0.01745	-0.0003	-0.999848	-0.01745	-0.0003	-0.99985	0.00	0.00	0.00

As you can see, I have fixed angle of (α) and have copied and pasted all angles on green, red, blue and yellow colors that if you change only angle of (α), you can easily get all the same directions just like below examples:

L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
		α	5										
		-0.99619	Cos(90+α)	-0.08716	Sin β	0.996195	Cos(180+α)	-0.99619	Sin θ	0.087156			
		-0.99619	Cos(90-α)	0.087156	Sin β'	0.996195	Cos α	0.996195	Sin θ'	-0.08716			
		175	β	95			θ'	5					
		175	β'	85			θ	185					
			β+180	275			θ'+180	185					
			β'+180	265			θ+180	365					

β	β'	θ'	θ	i	j	k	i	j	k	i·i	j·j	k·k
95	85	5	185	0.086824	0.007596	0.9961947	0.086824	0.007596	0.99619	0.00	0.00	0.00
95	85	185	5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.00	0.00
275	265	185	365	0.086824	0.007596	-0.996155	0.086824	0.007596	-0.99619	0.00	0.00	0.00
275	265	365	185	-0.08682	-0.0076	-0.996155	-0.08682	-0.0076	-0.99619	0.00	0.00	0.00

M	N	O	P	Q	R	S	T	U	V	W	X	Y
	α	7										
	-0.99255	Cos(90+α)	-0.12187	Sin β	0.992546	Cos(180+α)	-0.99255	Sin θ	0.121869			
	-0.99255	Cos(90-α)	0.121869	Sin β'	0.992546	Cos α	0.992546	Sin θ'	-0.12187			
	171	β	97			θ'	7					
	171	β'	83			θ	187					
		β+180	277			θ'+180	187					
		β'+180	263			θ+180	367					

β	β'	θ'	θ	i	j	k	i	j	k	i·i	j·j	k·k
97	83	7	187	0.120961	0.014851	0.9925462	0.120961	0.014851	0.992546	0.00	0.00	0.00
97	83	187	7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.00	0.00
277	263	187	367	0.120961	0.014852	-0.992546	0.120961	0.014851	-0.99255	0.00	0.00	0.00
277	263	367	187	-0.12096	-0.01485	-0.992546	-0.12096	-0.01485	-0.99255	0.00	0.00	0.00

You can find the results for all 360 degrees by using this algorithm and a sensitivity analysis between α and θ, θ', β, and β'.

Method (2):

If we apply all 360 degrees for algorithm method (1), we will have 180 states for θ and θ' and 181 states for β and β' where we can not find the angles mentioned in above example (θ = 104 and β = 253). It means that method (1) gives us incomplete results. But, by using the method mentioned in article of "Can We Solve a Nonlinear Equation with Many Variables?" posted on link: http://emfps.blogspot.co.uk/2016/10/can-we-solve-nonlinear-equation-with.html, we can generate the complete results as follows:

For condition (1), we have: $tg\ \beta = -\ tg\ \beta'$

For condition (2), we have: $tg\ \theta = tg\ \theta'$

For establishing condition (1), we have to solve below equation:

$tg\ \beta + tg\ \beta' = 0$

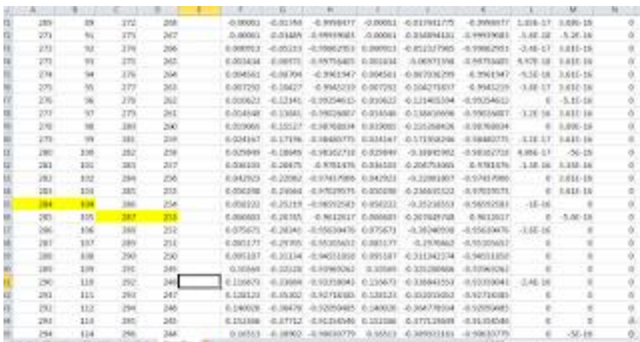
This is an equation with two independent variables which can be solved with the method mentioned in article of "Can We Solve a Nonlinear Equation with Many Variables?" posted on link: http://emfps.blogspot.co.uk/2016/10/can-we-solve-nonlinear-equation-with.html,.

For establishing condition (2), we have to solve below equation:

$tg\ \beta - tg\ \beta' = 0$

This is an equation with two independent variables which can be also solved with above method.

By using method (2), we will have 360 states for θ and θ' and 358 states for β and β' in which the angles mentioned in above example (θ = 104 and β = 253) are also included in our results just like below cited:



Finally, method (2) says to us that there are (360 * 357) / 2 = 64260 repetitions which should be deducted from the total 130322 states where we should take our analysis in accordance with 66062 states.

By analysis of the results extracted from the method (2), we can find the general formulas as follows:

$\boldsymbol{\vartheta' = \vartheta + 180 \quad \text{If} \quad 0 \le \vartheta \le 180}$

$\boldsymbol{\beta' = 180 - \beta \quad \text{If} \quad 0 \le \beta \le 180}$

$\boldsymbol{\vartheta' = \vartheta - 180 \quad \text{If} \quad 180 \le \vartheta \le 360}$

$\boldsymbol{\beta' = 540 - \beta \quad \text{If} \quad 180 \le \beta \le 360}$

θ and β = degree

Example (1):

Suppose θ = 56 degree and β = 112 degree. According to above formulas, we should use below equations:

$\theta' = \theta + 180 \quad \text{If} \quad 0 \le \theta \le 180$

$\beta' = 180 - \beta \quad \text{If} \quad 0 \le \beta \le 180$

Then we have:

$\theta' = 180 + 56 = 236$ degree

$\beta' = 180 - 112 = 68$ degree

By using equations mentioned in article of “The Change Depends on the Direction of the Motion: Generating All Directions in 3D Space” posted on link: https://emfps.blogspot.com/2017/05/the-change-depends-on-direction-of.html, we will have below directions:

Direction V =(cos β * cos ϑ) i + (cos β * sin ϑ) j + (sin β) k

For $\theta = 56$ and $\beta = 112$, Direction V is:

Direction V = -0.20948 i -0.31056 j + 0.927184 k

For $\theta' = 236$ and $\beta' = 68$, Direction V is:

Direction V = -0.20948 i -0.31056 j + 0.927184 k

You can see that the direction both of them are the same.

Example (2):

Suppose $\theta = 221$ degree and $\beta = 295$ degree. According to above formulas, we should use below equations:

$\theta' = \theta - 180$ If $180 \le \theta \le 360$

$\beta' = 540 - \beta$ If $180 \le \beta \le 360$

Then we have:

$\theta' = 221 - 180 = 41$ degree

$\beta' = 540 - 295 = 245$ degree

Direction V =(cos β * cos ϑ) i + (cos β * sin ϑ) j + (sin β) k

For $\theta = 221$ and $\beta = 295$, Direction V is:

Direction V = -0.31895 i -0.27727 j – 0.90631 k

For $\theta' = 41$ and $\beta' = 245$, Direction V is:

Direction V = -0.31895 i -0.27727 j – 0.90631 k

You can see that the direction both of them are the same.

In the reference with above formulas, we reach to a constant number equal to 64442 rows on spreadsheet of excel which shows us all directions in 3D space.

Conclusion

When we encounter the big data on our spreadsheet, the most crucial thing to bear in mind is to deduct the data in which there will not be any difference in our final results and analysis because it is possible that our laptop and computer will not be able to process the big data due to its technical characteristics.

The result of this article says to us that we can decrease 130321 rows on our spreadsheet to 64442 rows while the results and analysis will be finally the same.

Example:

Suppose you have a curve in 3D space as follows and the point P (x, y, z) wants to move from P0 (-1, 3, 2) on this curve toward all directions in amount of 4 unit (Δs = 4 unit). The question is:

What is the maximum change of below function when point P (x, y, z) moves in all directions? Which direction will the maximum change of the function occur?

Solution:

$$f(x,y,z) = x^2 + y^2 - 2z^2$$

$$\nabla f = -2i + 6j - 8k$$

$$\Delta f \cong \nabla f \,.\, U_i \,.\, \Delta s$$

$$1 \le i \le 64442 \,,\, i \in N$$

The answer is:

$$\Delta f_{max} = 40.79105 \ \textit{unit, in Direction}\ U = -0.19025i + 0.585529j - 0.78801k$$

Suppose above function is a curve of heat equation to analyze thermal conduction in a room. Why do I say, assume this function is a curve of heat equation? Because this function follows the Laplace's equation.

Now, when I try above formula for "i" between 1 and 130321, the answer is the same.

The Change Depends on the Direction of the Motion: The Gradient Vector and Symmetric Group Action (1)

https://emfps.blogspot.com/2017/08/the-change-depends-on-direction-of_27.html

Following to article of "The Change Depends on the Direction of the Motion: The Symmetric Group Action (2)" posted on link: http://www.emfps.org/2017/08/the-change-depends-on-direction-of_9.html?m=1, before I start new operators with four points, I would like to inform you that there are many other properties which can be derived from previous theorems. The purpose of this article is, to use the gradient as an operator accompanied by symmetric group action in which they work together. In this article as an example, I only examine the properties of an operator 3*3 which works with gradient vector of function:

f (x, y, z) = x^n + y^n + z^n

Regarding to my previous articles, I introduced to you many symmetric groups actions and 11 theorems where all of them accompanied by the gradient vector of any function will generate many properties and theorems.

As you saw, we had matrix "M" as below operator 3*3:

M =

$$\begin{pmatrix} r \cdot \cos\beta \cdot \cos\theta & r \cdot \cos\beta \cdot \sin\theta & r \cdot \sin\beta \\ -r \cdot \cos(60-\beta) \cdot \cos\theta & -r \cdot \cos(60-\beta) \cdot \sin\theta & r \cdot \sin(60-\beta) \\ -r \cdot \cos(60+\beta) \cdot \cos\theta & -r \cdot \cos(60+\beta) \cdot \sin\theta & -r \cdot \sin(60+\beta) \end{pmatrix}$$

The property of function: w = f (x, y, z) = x^2 + y^2 + z^2

The gradient vector of this function is:

$$\nabla f = 2x + 2y + 2z$$

Theorem 12: Maximum and minimum magnitude of the vector produced by operator M and the gradient vector of function f (x, y, z) = x^2 + y^2 + z^2 are obtained by below formulas:

$$Max\,|\nabla.M| = \sqrt{6}\,r_1 r_2$$

$$Min\,|\nabla.M| = 0.0113030579770455\,r_1 r_2$$

Where:

r1 = radius in operator M

r2 = radius of each point on surface or space in accordance with its polar coordinates

Theorem 13: *Maximum and minimum magnitude of the vector produced by operator M and the gradient vector of function f (x, y, z) = x^3 + y^3 + z^3 are obtained by below formulas:*

$$Max\ \left|\nabla.M\right| = 3r_1r_2^2$$

$$Min\ \left|\nabla.M\right| = 0.0195453224827526\ r_1r_2^2$$

Where:

r1 = radius in operator M

r2 = radius of each point on surface or space in accordance with its polar coordinates

Theorem 14: *Maximum and minimum magnitude of the vector produced by operator M and the gradient vector of function f (x, y, z) = x^4 + y^4 + z^4 are obtained by below formulas:*

$$Max\ \left|\nabla.M\right| = 4r_1r_2^3$$

$$Min\ \left|\nabla.M\right| = 0.0225321142991723\ r_1r_2^3$$

Where:

r1 = radius in operator M

r2 = radius of each point on surface or space in accordance with its polar coordinates

Theorem 15: Maximum and minimum magnitude of the vector produced by operator M and the gradient vector of function f (x, y, z) = x^5 + y^5 + z^5 are obtained by below formulas:

$$Max\ \left|\nabla.M\right| = 5r_1r_2^4$$

$$Min\ \left|\nabla.M\right| = 0.0216462806047336\ r_1r_2^4$$

Where:

r1 = radius in operator M

r2 = radius of each point on surface or space in accordance with its polar coordinates

The property of function: w = f (x, y, z) = x^n + y^n + z^n

As we as well as know, "n" time partial differential of this function will be calculated by using below formula:

$$\nabla^n f = n!\,(i + j + k)$$

Theorem 16: For function **w = f (x, y, z) = x^n + y^n + z^n and operator M, we have:**

$$\nabla^n f.M = 0$$

Example:

Suppose you have below function:

w = f (x, y, z) = x^5 + y^5 + z^5

And also you have below conditions for operator M:

r1 = 22.6

θ = 51 Degree

β = 13 degree

Then, operator M will be:

M =

$$\begin{vmatrix} 13.8581 & 17.1133 & 5.0838938 \\ -9.69982 & -11.9783 & 16.528594 \\ -4.1583 & -5.13507 & -21.612487 \end{vmatrix}$$

According to above formula, we have:

$$\nabla^n f = n!\,(i + j + k)$$

$$\nabla^n f = 120(i + j + k)$$

Therefore, we can see:

$$\nabla^n f . M = 0$$

In the reference with theorem 16, we can find a very interesting theorem as follows:

When we say n! , it means that we can consider it as a constant value for any vector:

V = c (i + j + k)

Theorem 17: Each vector V = c (i + j + k) multiplied by operator M will be equal zero.

$$\boldsymbol{\vec{V}.\ M = 0\ \ If\ \ \vec{V} = c\ (i + j + k)}$$

The Change Depends on the Direction of the Motion: The Symmetric Group Action (2)

https://emfps.blogspot.com/2017/08/the-change-depends-on-direction-of_9.html?m=1

Following to article of "The Change Depends on the Direction of the Motion: The Symmetric Group Action (1)" posted on link: https://emfps.blogspot.com/2017/08/the-change-depends-on-direction-of.html, the purpose of this article is to introduce some properties of operators and transformations which are formed by moving three points on circle and sphere.

But, before starting of this article, let me tell you more explanations about theorems mentioned in previous article as follows:

1. All theorems in previous article denote to get the maximum and minimum for vectors in all directions but if we need to have the maximum and minimum of each point on surface or space exchanged by operators and transformations, all equations should be changed as follows:

Theorem (1): |V|max = (2^0.5).r1.r2 and |V|min = 0

Theorem (4): |V|max = r2.max (z, r1)*(2^0.5) and

|V|min = r2.min (z, r1)*(2^0.5)

Where:

r1 = radius in operator or transformation matrix

r2 = radius of each point on surface or space in accordance with its polar coordinates

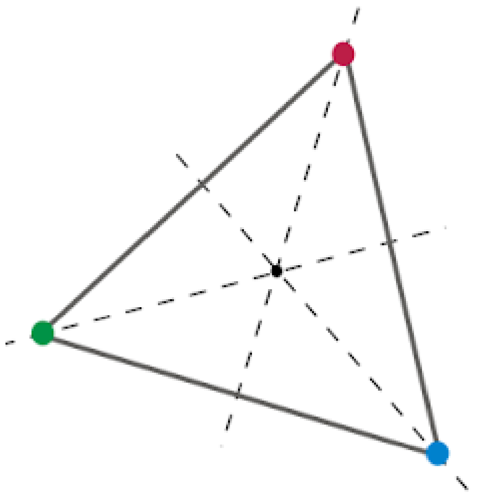
2. In Theorem (4), if z = r1 then we can say this transformation matrix maps a random point on surface to a random point on a sphere with radius equal to:

R = (2^0.5).r1.r2

R = radius of the sphere

An operator or transformation matrix formed by three points on circle

Suppose three points on a circle are rotating in which the distance among all three points are the same and equal just like below figure:



For reaching to above conditions, below polar coordinates for each point should be established:

A:

$x = r \cos \theta$

$y = r \sin \theta$

B:

$x = - r \sin (\theta + 30)$

$y = r \cos (\theta +30)$

C:

$x = - r \sin (30 - \theta)$

$y = - r \cos (30 - \theta)$

By considering any random number for “r” and “θ”, you can see not only all distances are equal but also all three points are on a circle.

Example:

r = 23   and   θ = 41 degree

AO = BO = CO = 23

AB = BC = CA = 39. 83716857

Above polar coordinates give us a transformation matrix 3*2 as follows:

$$M = \begin{pmatrix} r\cos\theta & r\sin\theta \\ -r\sin(\theta + 30) & r\cos(\theta + 30) \\ -r\sin(30 - \theta) & -r\cos(30 - \theta) \end{pmatrix}$$

The properties of transformation matrix 3*2 for R^3 to R^2

By multiplying matrix M by any 3D vectors in the space, we can extract the properties of this transformation matrix as follows:

Theorem (6): The maximum magnitude among 2D vectors produced by three point’s transformation matrix M is calculated by using below equation:

|V|max = 0.5. (6^0.5).r1.r2

Where:

r1 = radius in transformation matrix M

r2 = radius of each point in the space (3D) in accordance with its polar coordinates

The minimum magnitude is obtained by using below equation:

|V|min = r1.r2 / Φ

Φ = the constant coefficient equal to 176.943266509085

Here is a very interesting property:

Theorem (7): Always there are six points or six 2D vectors produced by three point's transformation matrix M which give us the maximum magnitude while there is only one point or one 2d vector which gives us the minimum magnitude in which the direction of all points or 2D vectors is between 0 degree to 180 degree.

The property of transformation matrix 2*3 for R^2 to R^3

It is transpose of above matrix in which we will have below matrix:

$$\begin{pmatrix} r\cos\theta & -r\sin(\theta + 30) & -r\sin(30-\theta) \\ r\sin\theta & r\cos(\theta+30) & -r\cos(30-\theta) \end{pmatrix}$$

Theorem (8): This transformation matrix maps a random point on surface to a random point on a sphere with radius equal to: R = 0.5. (6^0.5).r1.r2

Where:

R = radius of the sphere

r1 = radius in transformation matrix M

r2 = radius of each point on the surface (2D) in accordance with its polar coordinates

The properties of an operator 3*3

If we want to study these three points in 3D space rotating on circle or sphere, we will have an operator 3*3. In this case, there are several statements where I have started three forms as follows:

1. I added a constant coordinate (z) for each point and matrix will be:

$$M = \begin{pmatrix} r\cos\theta & r\sin\theta & z \\ -r\sin(\theta + 30) & r\cos(\theta + 30) & z \\ -r\sin(30 - \theta) & -r\cos(30 - \theta) & z \end{pmatrix}$$

Theorem (9): Maximum and minimum magnitudes among 3D vectors produced by operator M are calculated by using below equations and conditions:

If $r_1 / z > 2^{0.5}$ ***Then*** $|V|_{max} = 0.5.(6^{0.5}).r_1.r_2$ ***and***

$|V|_{min} = (3^{0.5}).r_2.\min(z, r_1)$

If $r_1 / z < 2^{0.5}$ ***Then*** $|V|_{max} = (3^{0.5}).r_2.\max(z, r_1)$ ***and***

$|V|_{min} = 0.5.(6^{0.5}).r_1.r_2$

Theorem (10): If $r_1 / z = 2^{0.5}$ ***then this operator maps a random point in the space to a random point on a sphere with radius equal to:***

$R = 0.5.(6^{0.5}).r_1.r_2$ ***or***

$R = (3^{0.5}).r_2.\min(z, r_1)$

Where:

R = radius of the sphere

r_1 ***= radius in operator M***

r_2 ***= radius of each point in the space (3D) in accordance with its polar coordinates***

Theorem (11): If $r1/z > 2^{0.5}$ Then, Always there are six points in the space or six 3D vectors produced by three point's operator M which give us the maximum magnitude while there is only one point or one 3D vector which gives us the minimum magnitude.

If $r1/z < 2^{0.5}$ Then, Always there are six points in the space or six 3D vectors produced by three point's operator M which give us the minimum magnitude while there is only one point or one 3D vector which gives us the maximum magnitude.

2. I replaced the constant coordinate (y) instead of (z):

$$\begin{pmatrix} r\cos\theta & y & r\sin\theta \\ -r\sin(\theta+30) & y & r\cos(\theta+30) \\ -r\sin(30-\theta) & y & -r\cos(30-\theta) \end{pmatrix}$$

The properties of this operator are similar to theorems (9), (10) and (11).

3. Suppose that three points in the space which have the same distance are rotating on a sphere. In this case, we will have below polar coordinates for all three points as follows:

Point A:

$x = r * \cos\beta * \cos\theta$

$y = r * \cos\beta * \sin\theta$

$z = r * \sin\beta$

Point B:

$x = -r * \cos(60 - \beta) * \cos\theta$

$y = -r * \cos(60 - \beta) * \sin\theta$

$z = r * \sin(60 - \beta)$

Point C:

$x = -r * \cos(60 + \beta) * \cos\theta$

$y = -r * \cos(60 + \beta) * \sin\theta$

$z = -r * \sin(60 + \beta)$

Example:

Assume $\theta = 56$, $\beta = 17$ and $r = 31$

According to above coordinates, we can calculate the distance among points and also the distance between points and center of sphere which answers are:

AO = BO = CO = 31

AB = BC = CA = 53.69358

The properties of operator 3*3

Above coordinates of points A and B and C make an operator 3*3 as follows:

$$\begin{pmatrix} r * \cos\beta * \cos\theta & r * \cos\beta * \sin\theta & r * \sin\beta \\ -r * \cos(60 - \beta) * \cos\theta & -r * \cos(60 - \beta) * \sin\theta & r * \sin(60 - \beta) \\ -r * \cos(60 + \beta) * \cos\theta & -r * \cos(60 + \beta) * \sin\theta & -r * \sin(60 + \beta) \end{pmatrix}$$

Note: ϑ and β have been introduced in my previous article of "The Change Depends on the Direction of the Motion: Generating All Directions in 3D Space" posted on link: https://emfps.blogspot.com/2017/05/the-change-depends-on-direction-of.html

The property of this transformation matrix is similar to theorems (6) and (7).

An operator or transformation matrix formed by four points on circle

To be continued….