MATLAB®

STUDENT VERSION

Computation

Visualization

Programming

The MATH WORKS Inc.

Learning MATLAB

Version 6 (Release 12)

How to Contact The MathWorks:

`www.mathworks.com`	Web
`ftp.mathworks.com`	Anonymous FTP server
`comp.soft-sys.matlab`	Newsgroup
`suggest@mathworks.com`	Product enhancement suggestions
`bugs@mathworks.com`	Bug reports
`doc@mathworks.com`	Documentation error reports

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Learning MATLAB

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Introduction

About the Student Version

MATLAB® & Simulink® are the premier software packages for technical computation, data analysis, and visualization in education and industry. The Student Version of MATLAB & Simulink provides all of the features of professional MATLAB, with no limitations, and the full functionality of professional Simulink, with model sizes up to 300 blocks. The Student Version gives you immediate access to the high-performance numeric computing power you need.

MATLAB allows you to focus on your course work and applications rather than on programming details. It enables you to solve many numerical problems in a fraction of the time it would take you to write a program in a lower level language. MATLAB helps you better understand and apply concepts in applications ranging from engineering and mathematics to chemistry, biology, and economics.

Simulink, included with the Student Version, provides a block diagram tool for modeling and simulating dynamical systems, including signal processing, controls, communications, and other complex systems.

The Symbolic Math Toolbox, also included with the Student Version, is based on the Maple® V symbolic kernel and lets you perform symbolic computations and variable-precision arithmetic.

MATLAB products are used in a broad range of industries, including automotive, aerospace, electronics, environmental, telecommunications, computer peripherals, finance, and medical. More than 400,000 technical professionals at the world's most innovative technology companies, government research labs, financial institutions, and at more than 2,000 universities rely on MATLAB and Simulink as the fundamental tools for their engineering and scientific work.

Student Use Policy

This Student License is for use in conjunction with courses offered at a degree-granting institution. The MathWorks offers this license as a special service to the student community and asks your help in seeing that its terms are not abused.

To use this Student License, you must be a student using the software in conjunction with courses offered at degree-granting institutions.

You may not use this Student License at a company or government lab. Also, you may not use it for research or for commercial or industrial purposes. In these cases, you can acquire the appropriate professional or academic version of the software by contacting The MathWorks.

Differences Between the Student Version and the Professional Version

MATLAB

This version of MATLAB provides full support for all language features as well as graphics, external interface and Application Program Interface support, and access to every other feature of the professional version of MATLAB.

Note MATLAB does not have a matrix size limitation in this Student Version.

MATLAB Differences. There are a few small differences between the Student Version and the professional version of MATLAB:

- The MATLAB prompt in the Student Version is

  `EDU>>`

- The window title bars include the words

  <Student Version>

- All printouts contain the footer

  `Student Version of MATLAB`

  This footer is not an option that can be turned off; it will always appear in your printouts.

Simulink

This Student Version contains the complete Simulink product, which is used with MATLAB to model, simulate, and analyze dynamical systems.

Simulink Differences.

- Models are limited to 300 blocks.
- The window title bars include the words

  <Student Version>

- All printouts contain the footer

  `Student Version of MATLAB`

  This footer is not an option that can be turned off; it will always appear in your printouts.

Note *Using Simulink*, which is accessible from the Help browser, contains all of the Simulink related information in the *Learning Simulink* book plus additional, advanced information.

Symbolic Math Toolbox

The Symbolic Math Toolbox included with this Student Version lets you use an important subset of Maple. You can access all of the functions in the professional version of the Symbolic Math Toolbox except `maple`, `mapleinit`, `mfun`, `mfunlist`, and `mhelp`. For a complete list of all the available functions, see Appendix B, “Symbolic Math Toolbox Quick Reference.”

Obtaining Additional MathWorks Products

Many college courses recommend MATLAB as their standard instructional software. In some cases, the courses may require particular toolboxes, blocksets, or other products. Many of these products are available for student use. You may purchase and download these additional products at special student prices from the MathWorks Store at `www.mathworks.com/store`.

Although many professional toolboxes are available at student prices from the MathWorks Store, *not* every one is available for student use. Some of the toolboxes you can purchase include:

- Communications
- Control System
- Fuzzy Logic
- Image Processing
- Neural Network
- Optimization
- Signal Processing
- Statistics
- Stateflow® (A demo version of Stateflow is included with your Student Version.)

For an up-to-date list of which toolboxes are available, visit the MathWorks Store.

Note The toolboxes that are available for the Student Version of MATLAB & Simulink have the same functionality as the full, professional versions. However, these student versions will *only* work with the Student Version. Likewise, the professional versions of the toolboxes will *not* work with the Student Version.

Getting Started with MATLAB

What I Want	What I Should Do
I need to install MATLAB.	See Chapter 2, "Installation," in this book.
I want to start MATLAB.	**(PC)** Your MathWorks documentation CD must be in your CD-ROM drive to start MATLAB. Double-click the MATLAB icon on your desktop. **(Linux)** Enter the `matlab` command.
I'm new to MATLAB and want to learn it quickly.	Start by reading Chapters 1 through 6 of *Learning MATLAB*. The most important things to learn are how to enter matrices, how to use the `:` (colon) operator, and how to invoke functions. You will also get a brief overview of graphics and programming in MATLAB. After you master the basics, you can access the rest of the documentation through the online help facility (Help).
I want to look at some samples of what you can do with MATLAB.	There are numerous demonstrations included with MATLAB. You can see the demos by selecting **Demos** from the **Help** menu. (Linux users type `demo` at the MATLAB prompt.) There are demos in mathematics, graphics, visualization, and much more. You also will find a large selection of demos at `www.mathworks.com/demos`.

Finding Reference Information

What I Want	What I Should Do
I want to know how to use a specific function.	Use the online help facility (Help). To access Help, use the command `helpbrowser` or use the **Help** menu. The MATLAB Function Reference is also available from Help in PDF format (under **Printable Documentation**) if you want to print out any of the function descriptions in high-quality form. **Note:** Your MathWorks documentation CD must be in your CD-ROM drive to access Help.
I want to find a function for a specific purpose but I don't know its name.	There are several choices: • See "MATLAB Quick Reference" in this book for a list of MATLAB functions. • From Help, peruse the MATLAB functions by Category or Alphabetically. • Use `lookfor` (e.g., `lookfor inverse`) from the command line. • Use Index or Search from Help.
I want to learn about a specific topic like sparse matrices, ordinary differential equations, or cell arrays.	Use Help to locate the appropriate sections in *Using MATLAB*.
I want to know what functions are available in a general area.	Use Help to see the Function Reference by Category, or see Appendix A, "MATLAB Quick Reference," in this book for a list of MATLAB functions. Help provides access to the reference pages for the hundreds of functions included with MATLAB.
I want to learn about the Symbolic Math Toolbox.	See Chapter 7, "Symbolic Math Toolbox," and Appendix B, "Symbolic Math Toolbox Quick Reference," in this book. For complete descriptions of the Symbolic Math Toolbox functions, use Help and select **Reference** from **Symbolic Math Toolbox**.

Troubleshooting and Other Resources

What I Want	What I Should Do
I have a MATLAB specific problem I want help with.	Visit the Technical Support section (`www.mathworks.com/support`) of the MathWorks Web site and search the Knowledge Base of problem solutions.
I want to report a bug or make a suggestion.	Use Help or send e-mail to `bugs@mathworks.com` or `suggest@mathworks.com`.

Documentation Library

Your Student Version of MATLAB & Simulink contains much more documentation than the two printed books, *Learning MATLAB* and *Learning Simulink*. On your CD is a personal reference library of every book and reference page distributed by The MathWorks. Access this documentation library from Help.

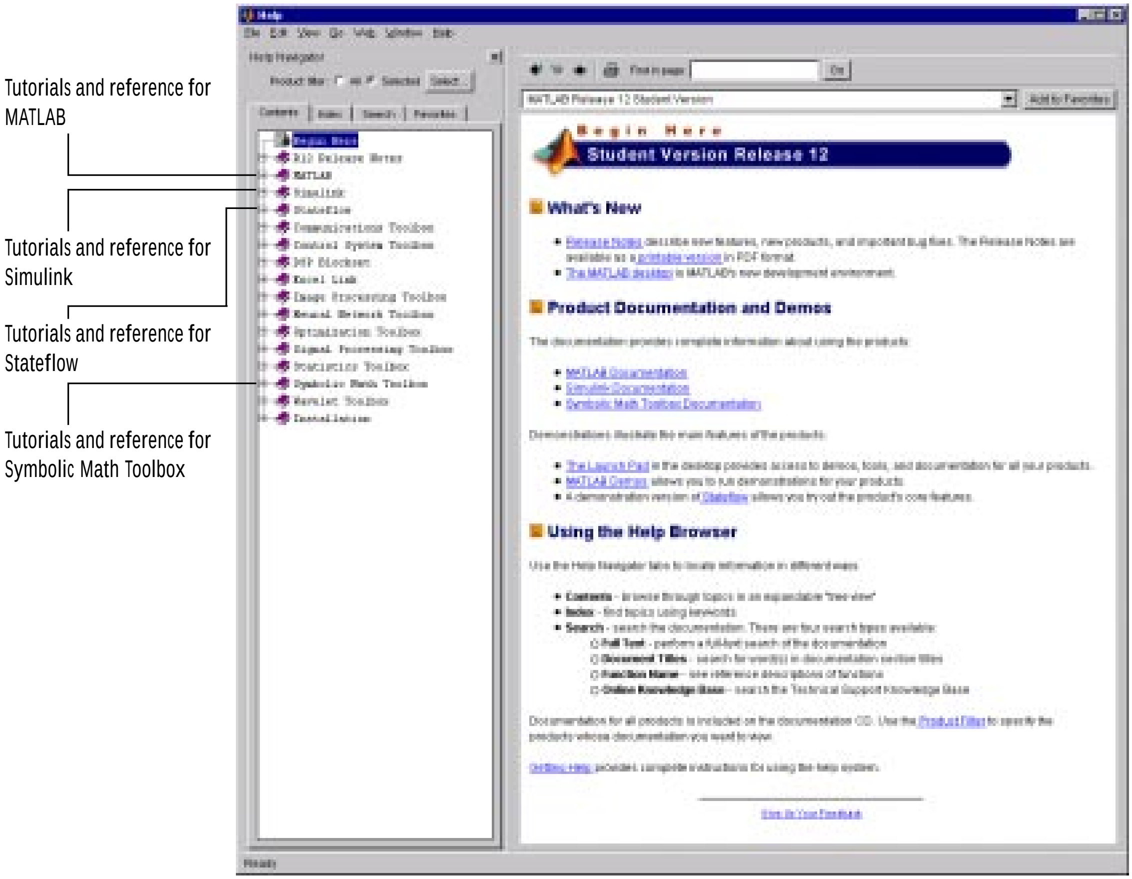
Note Even though you have the documentation set for the MathWorks family of products, not every product is available for the Student Version of MATLAB & Simulink. For an up-to-date list of available products, visit the MathWorks Store. At the store you can also purchase printed manuals for the MATLAB family of products.

Accessing the Online Documentation

Access the online documentation (Help) directly from your product CD. (Linux users should refer to Chapter 2, "Installation," for specific information on configuring and accessing the online Help from the CD.)

1. Place the CD in your CD-ROM drive.
2. Select **Full Product Family Help** from the **Help** menu.

Help appears in a separate window.



Note When you start MATLAB for the first time, the Help Navigator displays entries for additional products. To learn how to change the displayed product list, see the "Product Filter" on page 3-10.

MathWorks Web Site

Use your browser to visit the MathWorks Web site, `www.mathworks.com`. You'll find lots of information about MathWorks products and how they are used in education and industry, product demos, and MATLAB based books. From the Web site you will also be able to access our technical support resources, view a library of user and company supplied M-files, and get information about products and upcoming events.

MathWorks Education Web Site

This education-specific Web site, `www.mathworks.com/education`, contains many resources for various branches of engineering, mathematics, and science. Many of these include teaching examples, books, and other related products. You will also find a comprehensive list of links to Web sites where MATLAB is used for teaching and research at universities.

MATLAB Related Books

Hundreds of MATLAB related books are available from many different publishers. An up-to-date list is available at `www.mathworks.com/support/books`.

MathWorks Store

The MathWorks Store (`www.mathworks.com/store`) gives you an easy way to purchase add-on products and documentation.

Usenet Newsgroup

If you have access to Usenet newsgroups, you can join the active community of participants in the MATLAB specific group, `comp.soft-sys.matlab`. This forum is a gathering of professionals and students who use MATLAB and have questions or comments about it and its associated products. This is a great resource for posing questions and answering those of others. MathWorks staff also participates actively in this newsgroup.

MathWorks Knowledge Base

You can access the MathWorks Knowledge Base from the Support link on our Web site. Our Technical Support group maintains this database of frequently asked questions (FAQ). You can peruse the Knowledge Base to quickly locate relevant data. You will find numerous examples on graphics, mathematics, API, Simulink, and others. You can answer many of your questions by spending a few minutes with this around-the-clock resource.

Technical Support

The MathWorks does not provide telephone technical support to users of the Student Version of MATLAB & Simulink. There are numerous other vehicles of technical support that you can use. The Additional Sources of Information section in the CD holder identifies the ways to obtain support.

Registered users of the Student Version of MATLAB & Simulink can use our electronic technical support services to answer product questions. Visit our Technical Support Web site at `www.mathworks.com/support`.

After checking the available MathWorks sources for help, if you still cannot resolve your problem, you should contact your instructor. Your instructor should be able to help you, but if not, there is telephone technical support for registered instructors who have adopted the Student Version of MATLAB & Simulink in their courses.

Product Registration

Visit the MathWorks Web site (`www.mathworks.com/student`) and register your Student Version.

About MATLAB and Simulink

What Is MATLAB?

MATLAB is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses include:

- Math and computation
- Algorithm development
- Modeling, simulation, and prototyping
- Data analysis, exploration, and visualization
- Scientific and engineering graphics
- Application development, including graphical user interface building

MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. This allows you to solve many technical computing problems, especially those with matrix and vector formulations, in a fraction of the time it would take to write a program in a scalar noninteractive language such as C or Fortran.

The name MATLAB stands for *matrix laboratory*. MATLAB was originally written to provide easy access to matrix software developed by the LINPACK and EISPACK projects. Today, MATLAB uses software developed by the LAPACK and ARPACK projects, which together represent the state-of-the-art in software for matrix computation.

MATLAB has evolved over a period of years with input from many users. In university environments, it is the standard instructional tool for introductory and advanced courses in mathematics, engineering, and science. In industry, MATLAB is the tool of choice for high-productivity research, development, and analysis.

Toolboxes

MATLAB features a family of application-specific solutions called *toolboxes*. Very important to most users of MATLAB, toolboxes allow you to *learn* and *apply* specialized technology. Toolboxes are comprehensive collections of MATLAB functions (M-files) that extend the MATLAB environment to solve particular classes of problems. Areas in which toolboxes are available include signal processing, control systems, neural networks, fuzzy logic, wavelets, simulation, and many others.

The MATLAB System

The MATLAB system consists of five main parts:

Development Environment. This is the set of tools and facilities that help you use MATLAB functions and files. Many of these tools are graphical user interfaces. It includes the MATLAB desktop and Command Window, a command history, and browsers for viewing help, the workspace, files, and the search path.

The MATLAB Mathematical Function Library. This is a vast collection of computational algorithms ranging from elementary functions like sum, sine, cosine, and complex arithmetic, to more sophisticated functions like matrix inverse, matrix eigenvalues, Bessel functions, and fast Fourier transforms.

The MATLAB language. This is a high-level matrix/array language with control flow statements, functions, data structures, input/output, and object-oriented programming features. It allows both “programming in the small” to rapidly create quick and dirty throw-away programs, and “programming in the large” to create complete large and complex application programs.

Handle Graphics®. This is the MATLAB graphics system. It includes high-level commands for two-dimensional and three-dimensional data visualization, image processing, animation, and presentation graphics. It also includes low-level commands that allow you to fully customize the appearance of graphics as well as to build complete graphical user interfaces on your MATLAB applications.

The MATLAB Application Program Interface (API). This is a library that allows you to write C and Fortran programs that interact with MATLAB. It include facilities for calling routines from MATLAB (dynamic linking), calling MATLAB as a computational engine, and for reading and writing MAT-files.

What Is Simulink?

Simulink, a companion program to MATLAB, is an interactive system for simulating nonlinear dynamic systems. It is a graphical mouse-driven program that allows you to model a system by drawing a block diagram on the screen and manipulating it dynamically. It can work with linear, nonlinear, continuous-time, discrete-time, multirate, and hybrid systems.

Blocksets are add-ons to Simulink that provide additional libraries of blocks for specialized applications like communications, signal processing, and power systems.

Real-Time Workshop® is a program that allows you to generate C code from your block diagrams and to run it on a variety of real-time systems.

What Is Stateflow?

Stateflow is an interactive design tool for modeling and simulating complex reactive systems. Tightly integrated with Simulink and MATLAB, Stateflow provides Simulink users with an elegant solution for designing embedded systems by giving them an efficient way to incorporate complex control and supervisory logic within their Simulink models.

With Stateflow, you can quickly develop graphical models of event-driven systems using finite state machine theory, statechart formalisms, and flow diagram notation. Together, Stateflow and Simulink serve as an executable specification and virtual prototype of your system design.

Note Your Student Version of MATLAB & Simulink includes a comprehensive demo version of Stateflow.

2

Installation

Installing on Windows

System Requirements

Note For the most up-to-date information about system requirements, see the system requirements page, available in the Products area at the MathWorks Web site (`www.mathworks.com`).

MATLAB and Simulink

- Intel-based Pentium, Pentium Pro, Pentium II, Pentium III, or AMD Athlon personal computer
- Microsoft Windows 95, Windows 98, Windows 2000, Windows Me, or Windows NT 4.0 (with Service Pack 5 or 6a)
- CD-ROM drive for installation, program execution, and online documentation
- Disk space varies depending on size of partition. The MathWorks Installer will inform you of the disk space requirement for your particular partition.
- 64 MB RAM minimum; 128 MB RAM strongly recommended
- 8-bit graphics adapter and display (for 256 simultaneous colors)
- Netscape Navigator 4.0 or higher or Microsoft Internet Explorer 4.0 or higher is required.

Other recommended items include:

- Microsoft Windows supported graphics accelerator card
- Microsoft Windows supported printer
- Microsoft Windows supported sound card
- Microsoft Word 7.0 (Office 95), or 8.0 (Office 97), or Office 2000 is required to run the MATLAB Notebook.

Adobe Acrobat Reader is required to view and print the MATLAB online documentation that is in PDF format.

MEX-Files

MEX-files are dynamically linked subroutines that MATLAB can automatically load and execute. They provide a mechanism by which you can call your own C and Fortran subroutines from MATLAB as if they were built-in functions.

For More Information "External Interfaces/API" provides information on how to write MEX-files. "External Interfaces/API Reference" describes the collection of API functions. Both of these are available from Help.

If you plan to build your own MEX-files, one of the following is required:

- Borland C/C++ version 5.0 or 5.02
- Borland C++Builder version 3.0, 4.0, or 5.0
- Compaq Visual Fortran version 6.1
- DIGITAL Visual Fortran version 5.0
- Lcc C version 2.4 (included with MATLAB)
- Microsoft Visual C/C++ version 5.0 or 6.0
- Watcom C/C++ version 10.6 or 11

Note For an up-to-date list of all the compilers supported by MATLAB, see the MathWorks Technical Support Department's Technical Notes at:

`http://www.mathworks.com/support/tech-notes/v5/1600/1601.shtml`

Installing MATLAB

This list summarizes the steps in the standard installation procedure. You can perform the installation by simply following the instructions in the dialog boxes presented by the installation program; it walks you through this process.

1 Turn off any virus protection software you have running.

2 Exit any existing copies of MATLAB you have running.

3 Insert the MathWorks product CD into your CD-ROM drive. The installation program starts automatically when the CD-ROM drive is ready. You can also run `setup.exe` from the product CD.

4 Install the Microsoft Java Virtual Machine (JVM), if prompted. The MathWorks Installer requires the Microsoft JVM.

   Note: The Java installation requires a system reboot.

5 View the **Welcome** screen and review the Software License Agreement.

6 Review the Student Use Policy.

7 Enter your name and school name.

8 To install the complete set of software (MATLAB, Simulink, and the Symbolic Math Toolbox), make sure all of the components are selected in the **Product List** dialog box.

9 Specify the destination directory, that is, the directory where you want to save the files on your hard drive. To change directories, use the **Browse** button.

10 When the installation is complete, verify the installation by starting MATLAB and running one of the demo programs. To start MATLAB, double-click on the MATLAB icon that the installer creates on your desktop. To run the demo programs, select **Demos** from **Help**.

Note The MathWorks documentation CD must be in your CD-ROM drive to start MATLAB.

---

11 Customize any MATLAB environment options, if desired. For example, to include default definitions or any MATLAB expressions that you want executed every time MATLAB is invoked, create a file named `startup.m` in the `$MATLAB\toolbox\local` directory. MATLAB executes this file each time MATLAB is invoked.

12 Perform any additional configuration by typing the appropriate command at the MATLAB command prompt. For example, to configure the MATLAB Notebook, type `notebook -setup`. To configure a compiler to work with the MATLAB Application Program Interface, type `mex -setup`.

For More Information The *MATLAB Installation Guide for PC* provides additional installation information. This manual is available from Help.

Installing Additional Toolboxes

To purchase additional toolboxes, visit the MathWorks Store at (`www.mathworks.com/store`). Once you purchase a toolbox, it is downloaded to your computer.

When you download a toolbox, you receive an installation program for the toolbox. To install the toolbox, run the installation program by double-clicking on its icon. After you successfully install the toolbox, all of its functionality will be available to you when you start MATLAB.

Note Some toolboxes have `ReadMe` files associated with them. When you download the toolbox, check to see if there is a `ReadMe` file. These files contain important information about the toolbox and possibly installation and configuration notes. To view the `ReadMe` file for a toolbox, use the `whatsnew` command.

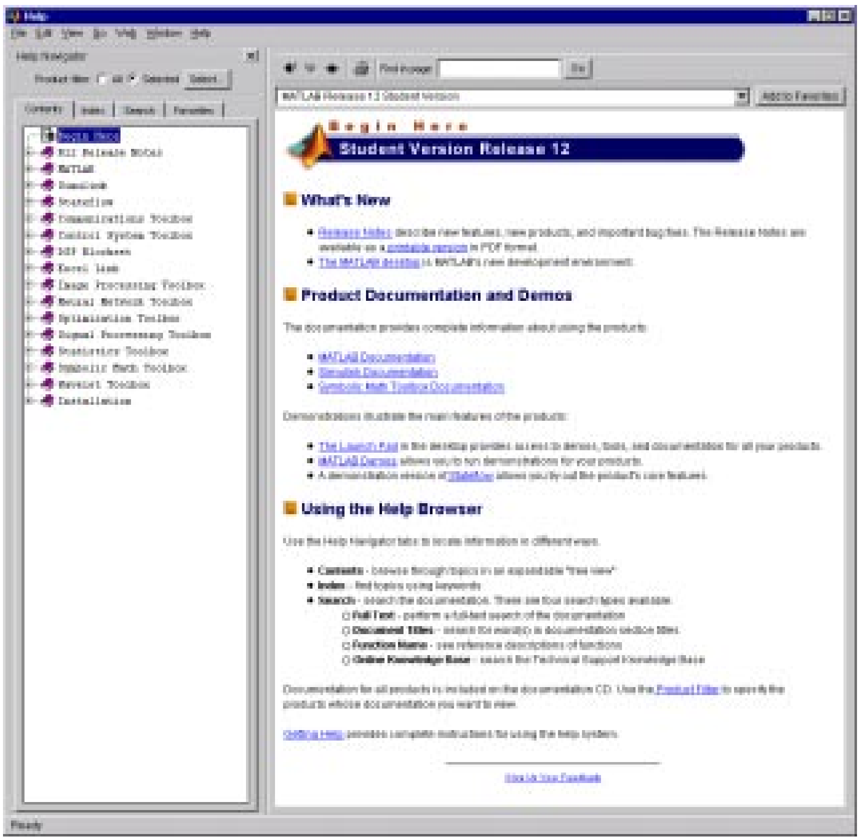
Accessing the Online Documentation (Help)

Access the online documentation (Help) directly from your documentation CD.

1 Place the documentation CD in your CD-ROM drive.

2 Select **Full Product Family Help** from the **Help** menu in the MATLAB Command Window. You can also type `helpbrowser` at the MATLAB prompt.

The Help browser appears.



Installing on Linux

Note The Student Version of MATLAB & Simulink for the Linux platform is only available in the US and Canada.

System Requirements

Note For the most up-to-date information about system requirements, see the system requirements page, available in the products area at the MathWorks Web site (`www.mathworks.com`).

MATLAB and Simulink

- Intel-based Pentium, Pentium Pro, Pentium II, Pentium III, or AMD Athlon personal computer
- Linux 2.2.x kernel
  - glibc 2.1.x (2.1.2 or higher recommended)
  - gcc 2.95.2 (gcc, g++, g77)
  - xFree86 3.3.x (3.3.6 or higher recommended)
- X Windows (X11R6)
- 110 MB free disk space for MATLAB, Simulink, and Symbolic Math Toolbox
- 64 MB memory, additional memory strongly recommended
- 64 MB swap space
- CD-ROM drive for installation and online documentation
- 8-bit graphics adapter and display (for 256 simultaneous colors)
- Netscape Navigator 4.0 or higher is required.

Adobe Acrobat Reader is required to view and print the MATLAB online documentation that is in PDF format.

MEX-Files

MEX-files are dynamically linked subroutines that MATLAB can automatically load and execute. They provide a mechanism by which you can call your own C and Fortran subroutines from MATLAB as if they were built-in functions.

For More Information "External Interfaces/API" provides information on how to write MEX-files. "External Interfaces/API Reference" describes the collection of API functions. Both of these are available from Help.

If you plan to build your own MEX-files, you need an ANSIC C compiler (e.g., the GNU C compiler, `gcc`).

Note For an up-to-date list of all the compilers supported by MATLAB, see the MathWorks Technical Support Department's Technical Notes at:

```
http://www.mathworks.com/support/tech-notes/v5/1600/1601.shtml
```

Installing MATLAB

The following instructions describe how to install the Student Version of MATLAB & Simulink on your computer.

Note It is recommended that you log in as `root` to perform your installation.

Installing the Software

To install the Student Version:

1 If your CD-ROM drive is not accessible to your operating system, you will need to create a directory to be the mount point for it.

```
mkdir /cdrom
```

2 Place the MathWorks product CD into the CD-ROM drive.

3 Execute the command to mount the CD-ROM drive on your system. For example,

```
# mount -t iso9660 /dev/cdrom /cdrom
```

should work on most systems. If your `/etc/fstab` file has a line similar to

```
/dev/cdrom /cdrom iso9660 noauto,ro,user,exec 0 0
```

then nonroot users can mount the CD-ROM using the simplified command

```
$ mount /cdrom
```

Note If the `exec` option is missing (as it often is by default, for security reasons), you will receive a "Permission denied" error when attempting to run the install script. To remedy this, either use the full mount command shown above (as `root`) or add the `exec` option to the file `/etc/fstab`.

---

4 Move to the installation location using the `cd` command. For example, if you are going to install into the location `/usr/local/matlab6`, use the commands

```
cd /usr/local
mkdir matlab6
cd matlab6
```

Subsequent instructions in this section refer to this directory as `$MATLAB`.

5 Run the CD install script.

```
/cdrom/install_glnx86.sh
```

The welcome screen appears. Select **OK** to proceed with the installation.

Note If you need additional help on any step during this installation process, click the **Help** button at the bottom of the dialog box.

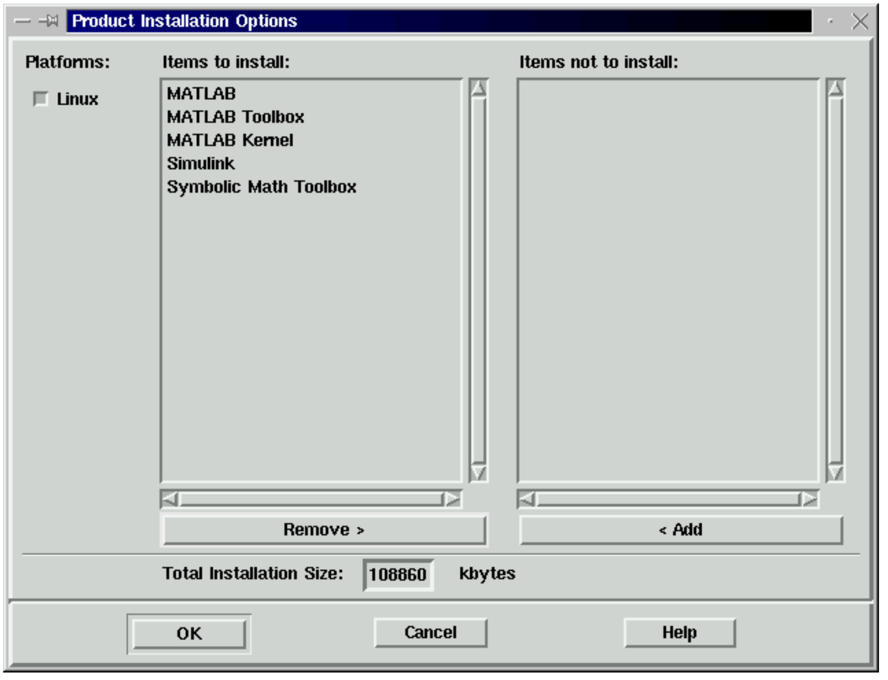
---

6 Accept or reject the software licensing agreement displayed. If you accept the terms of the agreement, you may proceed with the installation.

7 The **MATLAB Root Directory** screen is displayed. Select **OK** if the pathname for the MATLAB root directory is correct; otherwise, change it to the desired location.

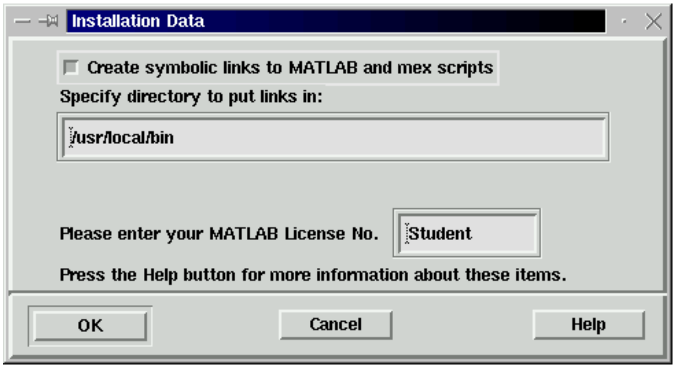
8 The system displays your license file. Press **OK**.

9 The installation program displays the **Product Installation Options** screen, which is similar to this.



The products you are licensed to install are listed in the **Items to install** list box. The right list box displays the products that you do not want to install. To install the complete Student Version of MATLAB & Simulink, you must install all the products for which you are licensed (MATLAB, MATLAB Toolbox, MATLAB Kernel, Simulink, and Symbolic Math Toolbox). Select **OK**.

10 The installation program displays the **Installation Data** screen.



Specify the directory location in your file system for symbolic links to the `matlab` and `mex` scripts. Choose a directory such as `/usr/local/bin`. You must be logged in as `root` to do this.

Select **OK** to continue.

11 The **Begin Product Installation** screen is displayed. Select **OK** to start the installation. After the installation is complete, the **Product Installation Complete** screen is displayed, assuming your installation is successful. Select **Exit** to exit from the setup program.

12 You must edit the `docopt.m` M-file located in the `$MATLAB/toolbox/local` directory to specify the path to the online documentation (Help). For example, if `/cdrom` is the path to your CD-ROM drive, then you would use `/cdrom/help`. To set the path using this example, change the lines in the `if isunix` block in the `docopt.m` file to

```
if isunix                    % UNIX
%   doccmd = '';
%   options = '';
    docpath = '/cdrom/help';
```

The `docopt.m` file also allows you to specify an alternative Web browser or additional initial browser options. It is configured for Netscape Navigator.

13 If desired, customize any MATLAB environment options. For example, to include default definitions or any MATLAB expressions that you want executed every time MATLAB is invoked, create a file named `startup.m` in the `$MATLAB/toolbox/local` directory. MATLAB executes this file each time MATLAB is invoked.

14 Start MATLAB by entering the `matlab` command. If you did not set up symbolic links in a directory on your path, type `$MATLAB/bin/matlab`.

Post Installation Procedures

Successful Installation

If you want to use the MATLAB Application Program Interface, you must configure the `mex` script to work with your compiler. Also, some toolboxes may require some additional configuration. For more information, see “Installing Additional Toolboxes” later in this section.

Unsuccessful Installation

If MATLAB does not execute correctly after installation:

1. Check the “R12 Release Notes” for the latest information concerning installation. This document is accessible from Help.

2. Repeat the installation procedure from the beginning but run the CD install script using the `-t` option.

   ```
   /cdrom/install_glnx86.sh -t
   ```

For More Information The *MATLAB Installation Guide for UNIX* provides additional installation information. This manual is available from Help.

Installing Additional Toolboxes

To purchase additional toolboxes, visit the MathWorks Store at (`www.mathworks.com/store`). Once you purchase a toolbox, it is downloaded to your computer. When you download a toolbox on Linux, you receive a tar file (a standard, compressed formatted file).

To install the toolbox, you must:

1 Place the tar file in `$MATLAB` and un-tar it.

```
tar -xf filename
```

2 Run `install`.

After you successfully install the toolbox, all of its functionality will be available to you when you start MATLAB.

Note Some toolboxes have `ReadMe` files associated with them. When you download the toolbox, check to see if there is a `ReadMe` file. These files contain important information about the toolbox and possibly installation and configuration notes. To view the `ReadMe` file for a toolbox, use the `whatsnew` command.

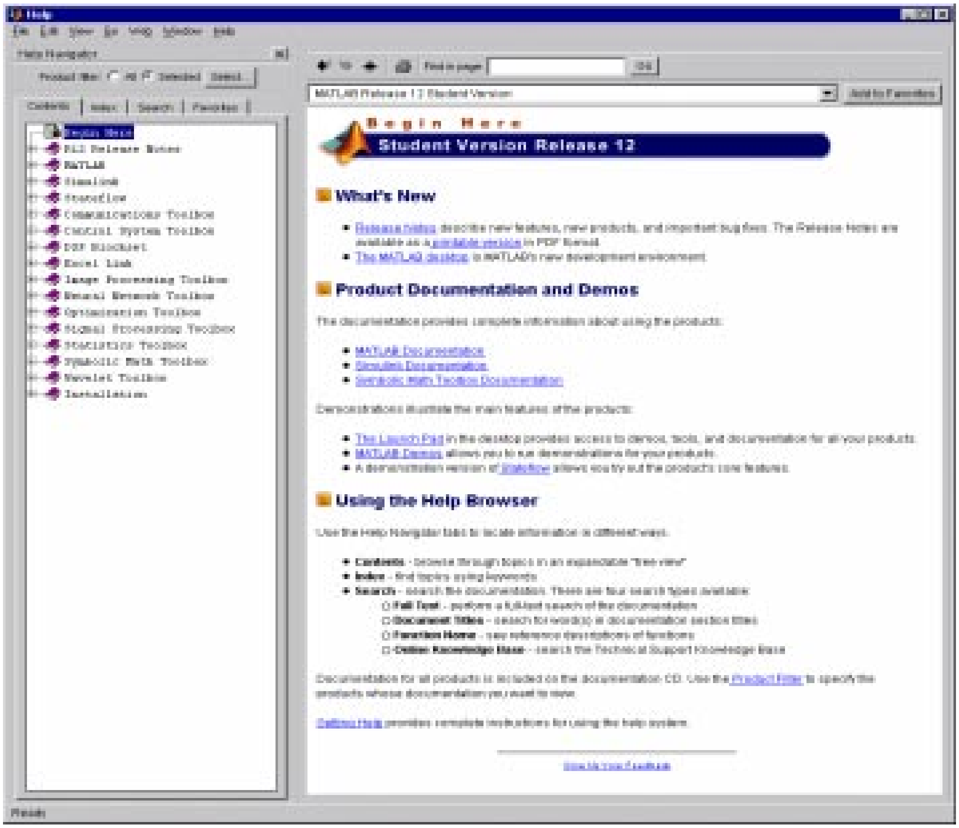
Accessing the Online Documentation (Help)

Access the online documentation (Help) directly from your documentation CD.

1 Place the documentation CD in your CD-ROM drive and mount it.

2 Select **Full Product Family Help** from the **Help** menu in the MATLAB Command Window. You can also type `helpbrowser` at the MATLAB prompt.

The Help browser appears.


Begin Here
Student Version Release 12
What's New
Product Documentation and Demos
Using the Help Browser

3

Development Environment

Introduction

This chapter provides a brief introduction to starting and quitting MATLAB, and the tools and functions that help you to work with MATLAB variables and files. For more information about the topics covered here, see the corresponding topics under “Development Environment” in the MATLAB documentation, which is available online.

Starting and Quitting MATLAB

Starting MATLAB

On a Microsoft Windows platform, to start MATLAB, double-click the MATLAB shortcut icon on your Windows desktop.

On Linux, to start MATLAB, type `matlab` at the operating system prompt.

Note On the Microsoft Windows platform, the documentation CD must be in your CD-ROM drive to start MATLAB. On both platforms, the documentation CD must be in your CD-ROM drive to access the online documentation.

After starting MATLAB, the MATLAB desktop opens – see “MATLAB Desktop” on page 3-4.

You can change the directory in which MATLAB starts, define startup options including running a script upon startup, and reduce startup time in some situations.

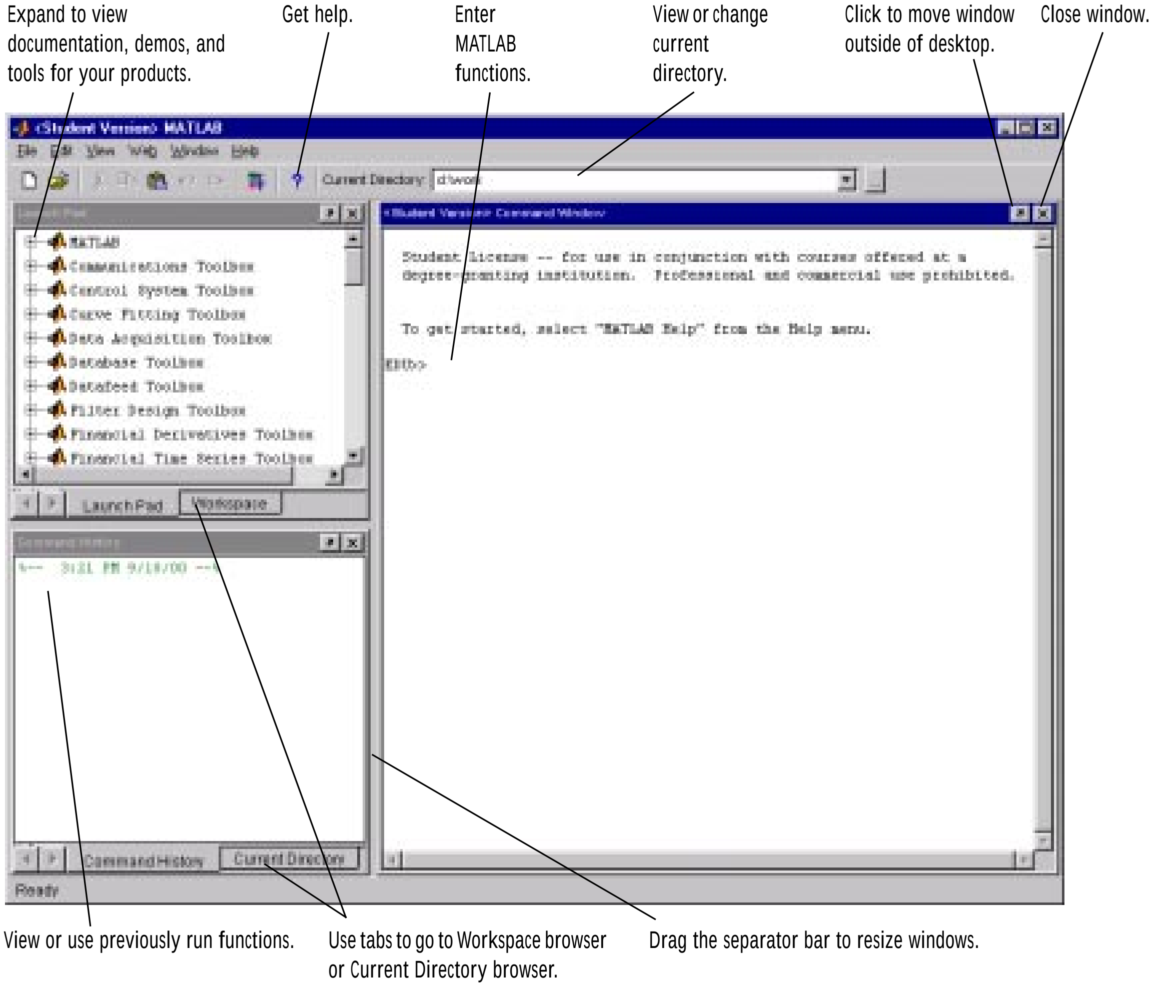
Quitting MATLAB

To end your MATLAB session, select **Exit MATLAB** from the **File** menu in the desktop, or type `quit` in the Command Window. To execute specified functions each time MATLAB quits, such as saving the workspace, you can create and run a `finish.m` script.

MATLAB Desktop

When you start MATLAB, the MATLAB desktop appears, containing tools (graphical user interfaces) for managing files, variables, and applications associated with MATLAB.

The first time MATLAB starts, the desktop appears as shown in the following illustration, although your Launch Pad may contain different entries.



You can change the way your desktop looks by opening, closing, moving, and resizing the tools in it. You can also move tools outside of the desktop or return them back inside the desktop (docking). All the desktop tools provide common features such as context menus and keyboard shortcuts.

You can specify certain characteristics for the desktop tools by selecting **Preferences** from the **File** menu. For example, you can specify the font characteristics for Command Window text. For more information, click the **Help** button in the **Preferences** dialog box.

Desktop Tools

This section provides an introduction to MATLAB's desktop tools. You can also use MATLAB functions to perform most of the features found in the desktop tools. The tools are:

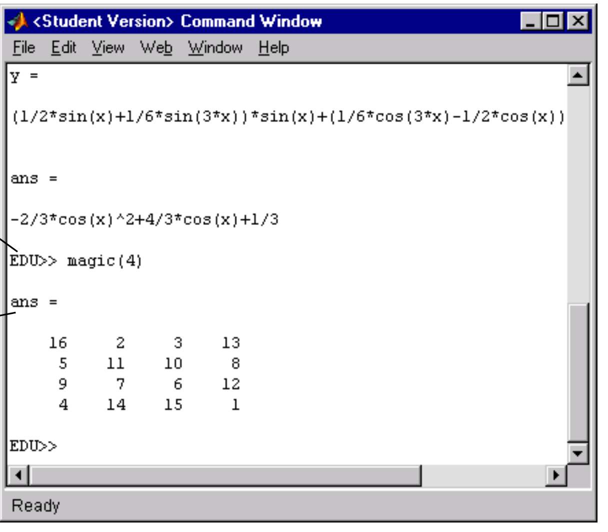
- "Command Window" on page 3-6
- "Command History" on page 3-7
- "Launch Pad" on page 3-8
- "Help Browser" on page 3-8
- "Current Directory Browser" on page 3-11
- "Workspace Browser" on page 3-12
- "Array Editor" on page 3-13
- "Editor/Debugger" on page 3-14

Command Window

Use the **Command Window** to enter variables and run functions and M-files. For more information on controlling input and output, see "Controlling Command Window Input and Output" on page 4-28.

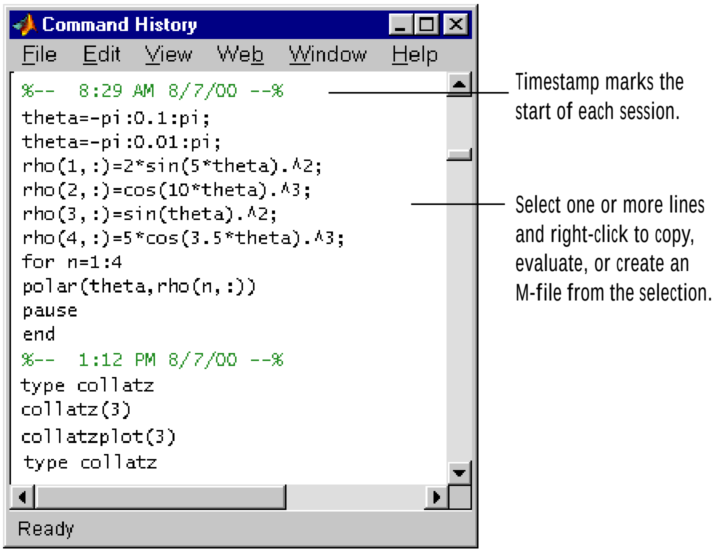
Type functions and variables at the MATLAB prompt.

MATLAB displays the results.



Command History

Lines you enter in the Command Window are logged in the **Command History** window. In the Command History, you can view previously used functions, and copy and execute selected lines.



To save the input and output from a MATLAB session to a file, use the `diary` function.

Note If other users share the same machine with you, using the same log in information, then they will have access to the functions you ran during a session via the Command History. If you do not want other users to have access to the Command History from your session, select **Clear Command History** from the **Edit** menu before you quit MATLAB.

Running External Programs

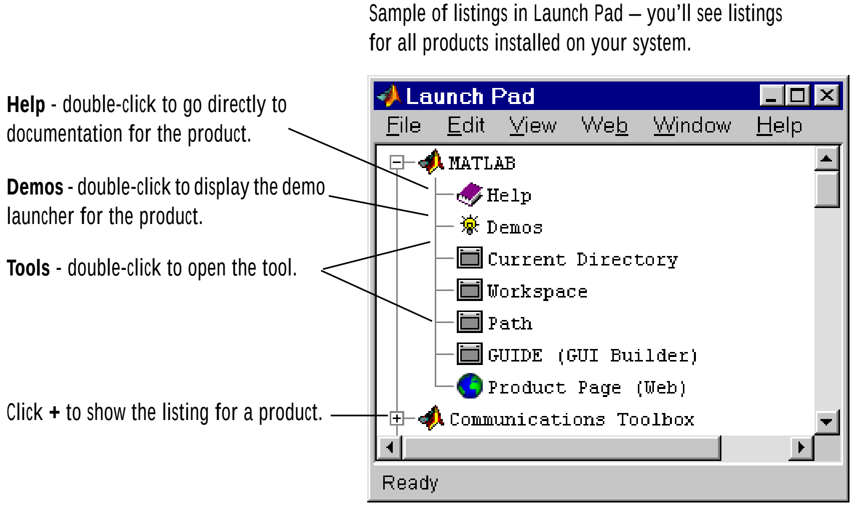
You can run external programs from the MATLAB Command Window. The exclamation point character ! is a shell escape and indicates that the rest of the input line is a command to the operating system. This is useful for invoking utilities or running other programs without quitting MATLAB. On Linux, for example,

```
!emacs magik.m
```

invokes an editor called `emacs` for a file named `magik.m`. When you quit the external program, the operating system returns control to MATLAB.

Launch Pad

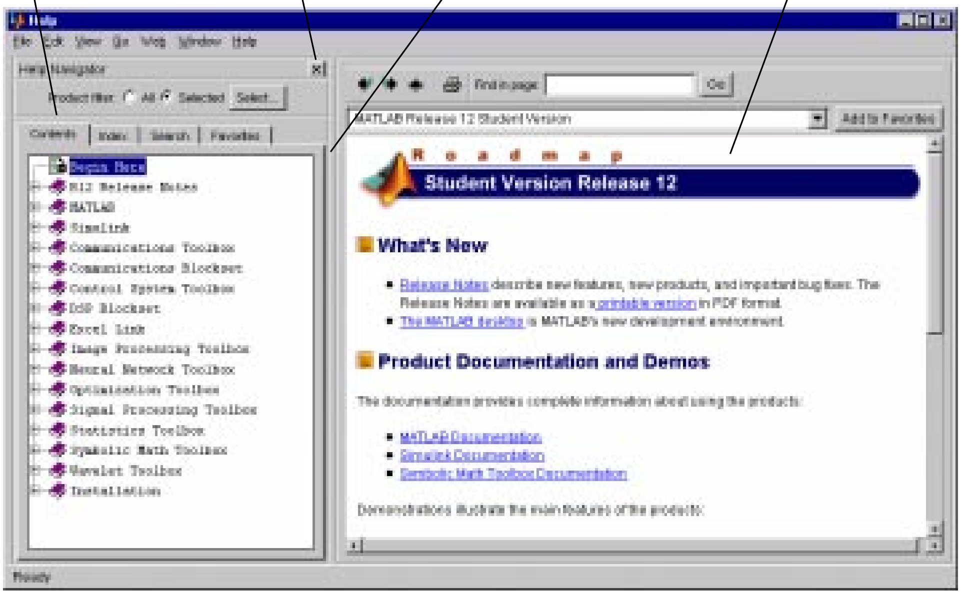
MATLAB's **Launch Pad** provides easy access to tools, demos, and documentation.



Help Browser

Use the Help browser to search and view documentation for all MathWorks products. The Help browser is a Web browser integrated into the MATLAB desktop that displays HTML documents.

To open the Help browser, click the help button ? in the toolbar, or type `helpbrowser` in the Command Window.



The Help browser consists of two panes, the Help Navigator, which you use to find information, and the display pane, where you view the information.

Help Navigator

Use to Help Navigator to find information. It includes:

- **Product filter** – Set the filter to show documentation only for the products you specify.

Note In the Student Version of MATLAB & Simulink, the product filter is initially set to display a subset of the entire documentation set. You can add or delete which product documentation is displayed by using the product filter.

- **Contents** tab – View the titles and tables of contents of documentation for your products.
- **Index** tab – Find specific index entries (selected keywords) in the MathWorks documentation for your products.
- **Search** tab – Look for a specific phrase in the documentation. To get help for a specific function, set the **Search type** to **Function Name**.
- **Favorites** tab – View a list of documents you previously designated as favorites.

Display Pane

After finding documentation using the Help Navigator, view it in the display pane. While viewing the documentation, you can:

- Browse to other pages – Use the arrows at the tops and bottoms of the pages, or use the back and forward buttons in the toolbar.
- Bookmark pages – Click the **Add to Favorites** button in the toolbar.
- Print pages – Click the print button in the toolbar.
- Find a term in the page – Type a term in the **Find in page** field in the toolbar and click **Go**.

Other features available in the display pane are: copying information, evaluating a selection, and viewing Web pages.

For More Help

In addition to the Help browser, you can use help functions. To get help for a specific function, use `doc`. For example, `doc format` displays help for the `format` function in the Help browser. Other means for getting help include contacting Technical Support (`http://www.mathworks.com/support`) and participating in the newsgroup for MATLAB users, `comp.soft-sys.matlab`.

Current Directory Browser

MATLAB file operations use the current directory and the search path as reference points. Any file you want to run must either be in the current directory or on the search path.

A quick way to view or change the current directory is by using the **Current Directory** field in the desktop toolbar as shown below.

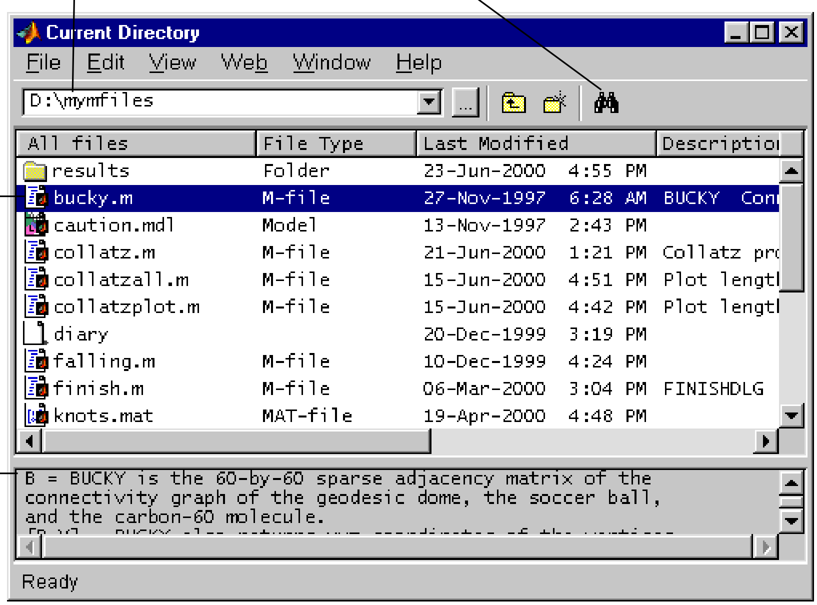
To search for, view, open, and make changes to MATLAB-related directories and files, use the MATLAB Current Directory browser. Alternatively, you can use the functions `dir`, `cd`, and `delete`.

Use the pathname edit box to view directories and their contents.

Click the find button to search for content within M-files.

Double-click a file to open it in an appropriate tool.

View the help portion of the selected M-file.



Search Path

To determine how to execute functions you call, MATLAB uses a *search path* to find M-files and other MATLAB-related files, which are organized in directories on your file system. Any file you want to run in MATLAB must reside in the current directory or in a directory that is on the search path. By default, the files supplied with MATLAB and MathWorks toolboxes are included in the search path.

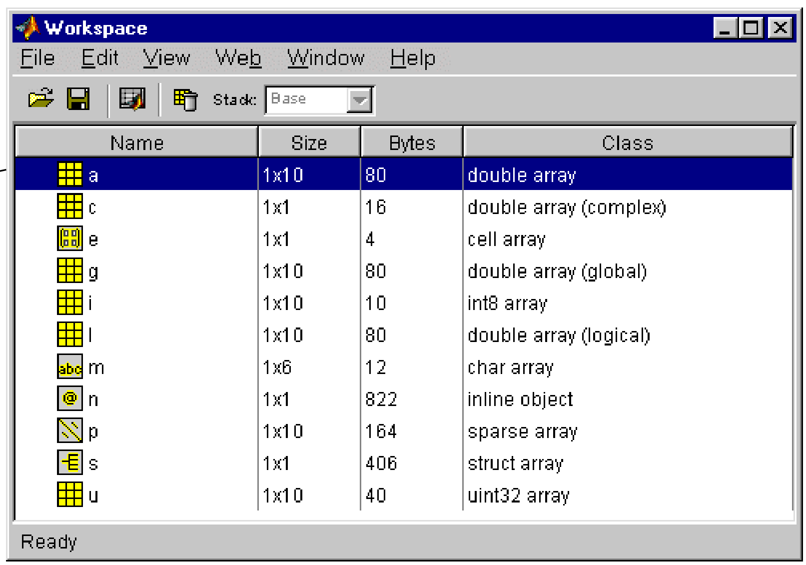
To see which directories are on the search path or to change the search path, select **Set Path** from the **File** menu in the desktop, and use the **Set Path** dialog box. Alternatively, you can use the `path` function to view the search path, `addpath` to add directories to the path, and `rmpath` to remove directories from the path.

Workspace Browser

The MATLAB workspace consists of the set of variables (named arrays) built up during a MATLAB session and stored in memory. You add variables to the workspace by using functions, running M-files, and loading saved workspaces.

To view the workspace and information about each variable, use the Workspace browser, or use the functions `who` and `whos`.

Double-click a variable to see and change its contents in the Array Editor.

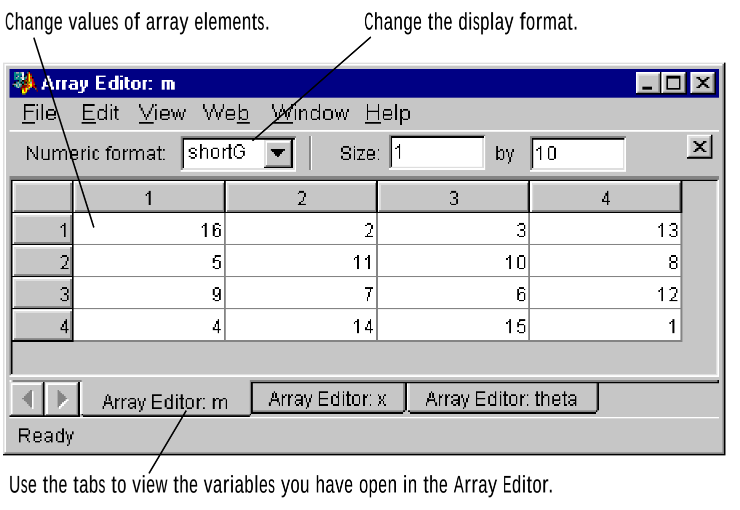


To delete variables from the workspace, select the variable and select **Delete** from the **Edit** menu. Alternatively, use the `clear` function.

The workspace is not maintained after you end the MATLAB session. To save the workspace to a file that can be read during a later MATLAB session, select **Save Workspace As** from the **File** menu, or use the `save` function. This saves the workspace to a binary file called a MAT-file, which has a `.mat` extension. There are options for saving to different formats. To read in a MAT-file, select **Import Data** from the **File** menu, or use the `load` function.

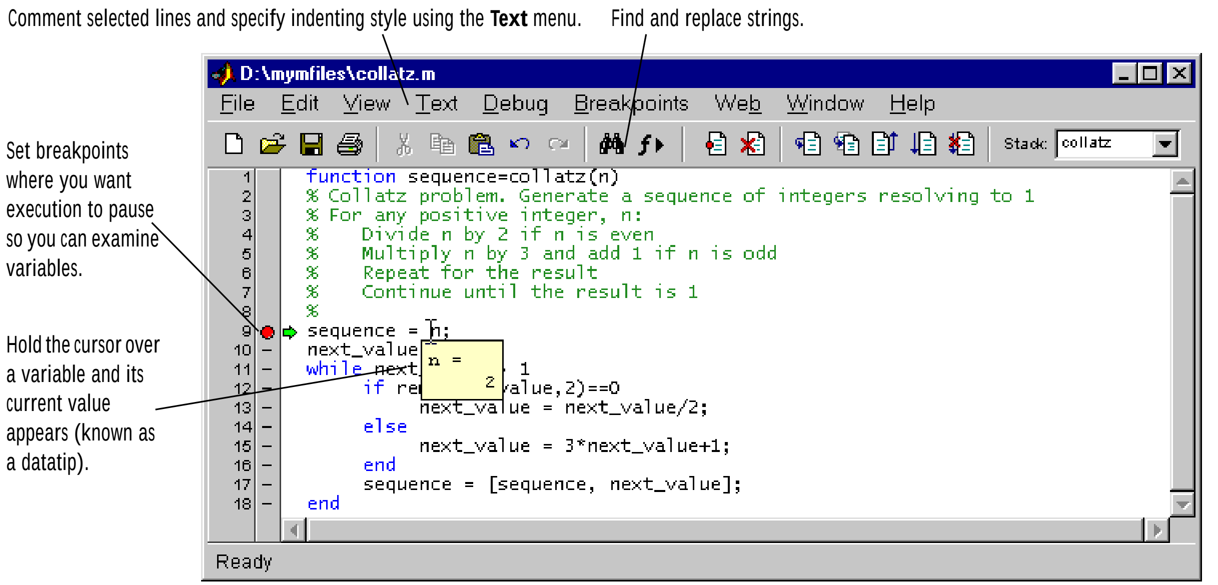
Array Editor

Double-click on a variable in the Workspace browser to see it in the Array Editor. Use the Array Editor to view and edit a visual representation of one- or two-dimensional numeric arrays, strings, and cell arrays of strings that are in the workspace.



Editor/Debugger

Use the Editor/Debugger to create and debug M-files, which are programs you write to run MATLAB functions. The Editor/Debugger provides a graphical user interface for basic text editing, as well as for M-file debugging.



You can use any text editor to create M-files, such as Emacs, and can use preferences (accessible from the desktop **File** menu) to specify that editor as the default. If you use another editor, you can still use the MATLAB Editor/Debugger for debugging, or you can use debugging functions, such as `dbstop`, which sets a breakpoint.

If you just need to view the contents of an M-file, you can display it in the Command Window by using the `type` function.

Other Development Environment Features

Additional development environment features are:

- Importing and Exporting Data – Techniques for bringing data created by other applications into the MATLAB workspace, including the Import Wizard, and packaging MATLAB workspace variables for use by other applications.
- Improving M-File Performance – The Profiler is a tool that measures where an M-file is spending its time. Use it to help you make speed improvements.
- Interfacing with Source Control Systems – Access your source control system from within MATLAB, Simulink, and Stateflow.
- Using Notebook – Access MATLAB's numeric computation and visualization software from within a word processing environment (Microsoft Word).

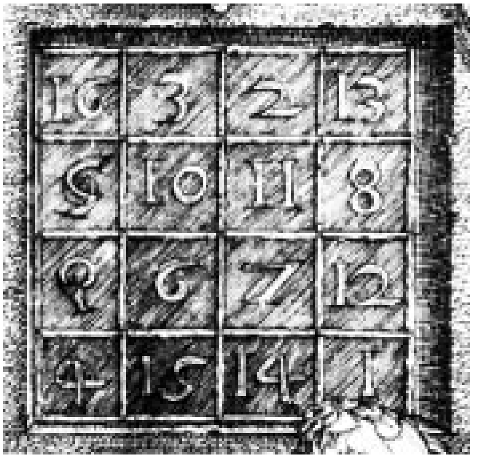
4

Getting Started

Matrices and Magic Squares

In MATLAB, a matrix is a rectangular array of numbers. Special meaning is sometimes attached to 1-by-1 matrices, which are scalars, and to matrices with only one row or column, which are vectors. MATLAB has other ways of storing both numeric and nonnumeric data, but in the beginning, it is usually best to think of everything as a matrix. The operations in MATLAB are designed to be as natural as possible. Where other programming languages work with numbers one at a time, MATLAB allows you to work with entire matrices quickly and easily. A good example matrix, used throughout this book, appears in the Renaissance engraving Melancholia I by the German artist and amateur mathematician Albrecht Dürer.

This image is filled with mathematical symbolism, and if you look carefully, you will see a matrix in the upper right corner. This matrix is known as a magic square and was believed by many in Dürer's time to have genuinely magical properties. It does turn out to have some fascinating characteristics worth exploring.



Entering Matrices

The best way for you to get started with MATLAB is to learn how to handle matrices. Start MATLAB and follow along with each example.

You can enter matrices into MATLAB in several different ways:

- Enter an explicit list of elements.
- Load matrices from external data files.
- Generate matrices using built-in functions.
- Create matrices with your own functions in M-files.

Start by entering Dürer's matrix as a list of its elements. You have only to follow a few basic conventions:

- Separate the elements of a row with blanks or commas.
- Use a semicolon, `;` , to indicate the end of each row.
- Surround the entire list of elements with square brackets, `[ ]`.

To enter Dürer's matrix, simply type in the Command Window

```
A = [16 3 2 13; 5 10 11 8; 9 6 7 12; 4 15 14 1]
```

MATLAB displays the matrix you just entered,

```
A =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    15    14     1
```

This exactly matches the numbers in the engraving. Once you have entered the matrix, it is automatically remembered in the MATLAB workspace. You can refer to it simply as A. Now that you have A in the workspace, take a look at what makes it so interesting. Why is it magic?

sum, transpose, and diag

You're probably already aware that the special properties of a magic square have to do with the various ways of summing its elements. If you take the sum along any row or column, or along either of the two main diagonals, you will always get the same number. Let's verify that using MATLAB. The first statement to try is

```
sum(A)
```

MATLAB replies with

```
ans =
    34    34    34    34
```

When you don't specify an output variable, MATLAB uses the variable ans, short for *answer*, to store the results of a calculation. You have computed a row vector containing the sums of the columns of A. Sure enough, each of the columns has the same sum, the *magic* sum, 34.

How about the row sums? MATLAB has a preference for working with the columns of a matrix, so the easiest way to get the row sums is to transpose the matrix, compute the column sums of the transpose, and then transpose the result. The transpose operation is denoted by an apostrophe or single quote, '. It flips a matrix about its main diagonal and it turns a row vector into a column vector. So

```
A'
```

produces

```
ans =
    16     5     9     4
     3    10     6    15
     2    11     7    14
    13     8    12     1
```

And

```
sum(A')'
```

produces a column vector containing the row sums

```
ans =
    34
    34
    34
    34
```

The sum of the elements on the main diagonal is easily obtained with the help of the `diag` function, which picks off that diagonal.

```
diag(A)
```

produces

```
ans =
    16
    10
     7
     1
```

and

```
sum(diag(A))
```

produces

```
ans =
    34
```

The other diagonal, the so-called *antidiagonal*, is not so important mathematically, so MATLAB does not have a ready-made function for it. But a function originally intended for use in graphics, `fliplr`, flips a matrix from left to right.

```
sum(diag(fliplr(A)))

ans =
    34
```

You have verified that the matrix in Dürer's engraving is indeed a magic square and, in the process, have sampled a few MATLAB matrix operations. The following sections continue to use this matrix to illustrate additional MATLAB capabilities.

Subscripts

The element in row `i` and column `j` of `A` is denoted by `A(i,j)`. For example, `A(4,2)` is the number in the fourth row and second column. For our magic square, `A(4,2)` is `15`. So it is possible to compute the sum of the elements in the fourth column of `A` by typing

```
A(1,4) + A(2,4) + A(3,4) + A(4,4)
```

This produces

```
ans =
     34
```

but is not the most elegant way of summing a single column.

It is also possible to refer to the elements of a matrix with a single subscript, `A(k)`. This is the usual way of referencing row and column vectors. But it can also apply to a fully two-dimensional matrix, in which case the array is regarded as one long column vector formed from the columns of the original matrix. So, for our magic square, `A(8)` is another way of referring to the value `15` stored in `A(4,2)`.

If you try to use the value of an element outside of the matrix, it is an error.

```
t = A(4,5)
Index exceeds matrix dimensions.
```

On the other hand, if you store a value in an element outside of the matrix, the size increases to accommodate the newcomer.

```
X = A;
X(4,5) = 17
```

```
X =
    16     3     2    13     0
     5    10    11     8     0
     9     6     7    12     0
     4    15    14     1    17
```

The Colon Operator

The colon, `:`, is one of MATLAB's most important operators. It occurs in several different forms. The expression

```
1:10
```

is a row vector containing the integers from 1 to 10

```
1     2     3     4     5     6     7     8     9    10
```

To obtain nonunit spacing, specify an increment. For example,

```
100:-7:50
```

is

```
100    93    86    79    72    65    58    51
```

and

```
0:pi/4:pi
```

is

```
0    0.7854    1.5708    2.3562    3.1416
```

Subscript expressions involving colons refer to portions of a matrix.

```
A(1:k,j)
```

is the first `k` elements of the `j`th column of `A`. So

```
sum(A(1:4,4))
```

computes the sum of the fourth column. But there is a better way. The colon by itself refers to *all* the elements in a row or column of a matrix and the keyword `end` refers to the *last* row or column. So

```
sum(A(:,end))
```

computes the sum of the elements in the last column of A.

```
ans =
      34
```

Why is the magic sum for a 4-by-4 square equal to 34? If the integers from 1 to 16 are sorted into four groups with equal sums, that sum must be

```
sum(1:16)/4
```

which, of course, is

```
ans =
      34
```

Using the Symbolic Math Toolbox, you can discover that the magic sum for an n-by-n magic square is $(n^3 + n)/2$.

The magic Function

MATLAB actually has a built-in function that creates magic squares of almost any size. Not surprisingly, this function is named magic.

```
B = magic(4)

B =
    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1
```

This matrix is almost the same as the one in the Dürer engraving and has all the same "magic" properties; the only difference is that the two middle columns are exchanged. To make this B into Dürer's A, swap the two middle columns.

```
A = B(:,[1 3 2 4])
```

This says "for each of the rows of matrix `B`, reorder the elements in the order 1, 3, 2, 4." It produces

```
A =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    15    14     1
```

Why would Dürer go to the trouble of rearranging the columns when he could have used MATLAB's ordering? No doubt he wanted to include the date of the engraving, 1514, at the bottom of his magic square.

For More Information "Using MATLAB," which is accessible from Help, provides comprehensive material on the development environment, mathematics, programming and data types, graphics, 3-D visualization, external interfaces/API, and creating graphical user interfaces in MATLAB.

Expressions

Like most other programming languages, MATLAB provides mathematical *expressions*, but unlike most programming languages, these expressions involve entire matrices. The building blocks of expressions are:

- Variables
- Numbers
- Operators
- Functions

Variables

MATLAB does not require any type declarations or dimension statements. When MATLAB encounters a new variable name, it automatically creates the variable and allocates the appropriate amount of storage. If the variable already exists, MATLAB changes its contents and, if necessary, allocates new storage. For example,

```
num_students = 25
```

creates a 1-by-1 matrix named `num_students` and stores the value 25 in its single element.

Variable names consist of a letter, followed by any number of letters, digits, or underscores. MATLAB uses only the first 31 characters of a variable name. MATLAB is case sensitive; it distinguishes between uppercase and lowercase letters. `A` and `a` are *not* the same variable. To view the matrix assigned to any variable, simply enter the variable name.

Numbers

MATLAB uses conventional decimal notation, with an optional decimal point and leading plus or minus sign, for numbers. *Scientific notation* uses the letter `e` to specify a power-of-ten scale factor. *Imaginary numbers* use either `i` or `j` as a suffix. Some examples of legal numbers are

```
3              -99            0.0001
9.6397238      1.60210e-20    6.02252e23
1i             -3.14159j      3e5i
```

All numbers are stored internally using the *long* format specified by the IEEE floating-point standard. Floating-point numbers have a finite *precision* of roughly 16 significant decimal digits and a finite *range* of roughly 10^{-308} to 10^{+308}.

Operators

Expressions use familiar arithmetic operators and precedence rules.

`+`	Addition
`-`	Subtraction
`*`	Multiplication
`/`	Division
`\`	Left division (described in “Matrices and Linear Algebra” in *Using MATLAB*)
`^`	Power
`'`	Complex conjugate transpose
`( )`	Specify evaluation order

Functions

MATLAB provides a large number of standard elementary mathematical functions, including `abs`, `sqrt`, `exp`, and `sin`. Taking the square root or logarithm of a negative number is not an error; the appropriate complex result is produced automatically. MATLAB also provides many more advanced mathematical functions, including Bessel and gamma functions. Most of these functions accept complex arguments. For a list of the elementary mathematical functions, type

```
help elfun
```

For a list of more advanced mathematical and matrix functions, type

```
help specfun
help elmat
```

For More Information Appendix A, "MATLAB Quick Reference," contains brief descriptions of the MATLAB functions. Use Help to access complete descriptions of all the MATLAB functions by category or alphabetically.

Some of the functions, like `sqrt` and `sin`, are *built-in*. They are part of the MATLAB core so they are very efficient, but the computational details are not readily accessible. Other functions, like `gamma` and `sinh`, are implemented in M-files. You can see the code and even modify it if you want.

Several special functions provide values of useful constants.

`pi`	3.14159265…
`i`	Imaginary unit, $\sqrt{-1}$
`j`	Same as `i`
`eps`	Floating-point relative precision, 2^{-52}
`realmin`	Smallest floating-point number, 2^{-1022}
`realmax`	Largest floating-point number, $(2-\varepsilon)2^{1023}$
`Inf`	Infinity
`NaN`	Not-a-number

Infinity is generated by dividing a nonzero value by zero, or by evaluating well defined mathematical expressions that *overflow*, i.e., exceed `realmax`. Not-a-number is generated by trying to evaluate expressions like `0/0` or `Inf-Inf` that do not have well defined mathematical values.

The function names are not reserved. It is possible to overwrite any of them with a new variable, such as

```
eps = 1.e-6
```

and then use that value in subsequent calculations. The original function can be restored with

```
clear eps
```

Examples of Expressions

You have already seen several examples of MATLAB expressions. Here are a few more examples, and the resulting values.

```
rho = (1+sqrt(5))/2
rho =
    1.6180

a = abs(3+4i)
a =
     5

z = sqrt(besselk(4/3,rho-i))
z =
   0.3730+ 0.3214i

huge = exp(log(realmax))
huge =
  1.7977e+308

toobig = pi*huge
toobig =
   Inf
```

Working with Matrices

This section introduces you to other ways of creating matrices.

Generating Matrices

MATLAB provides four functions that generate basic matrices.

`zeros`	All zeros
`ones`	All ones
`rand`	Uniformly distributed random elements
`randn`	Normally distributed random elements

Here are some examples.

```
Z = zeros(2,4)
Z =
     0     0     0     0
     0     0     0     0

F = 5*ones(3,3)
F =
     5     5     5
     5     5     5
     5     5     5

N = fix(10*rand(1,10))
N =
     4     9     4     4     8     5     2     6     8     0

R = randn(4,4)
R =
    1.0668    0.2944   -0.6918   -1.4410
    0.0593   -1.3362    0.8580    0.5711
   -0.0956    0.7143    1.2540   -0.3999
   -0.8323    1.6236   -1.5937    0.6900
```

The load Command

The `load` command reads binary files containing matrices generated by earlier MATLAB sessions, or reads text files containing numeric data. The text file should be organized as a rectangular table of numbers, separated by blanks, with one row per line, and an equal number of elements in each row. For example, outside of MATLAB, create a text file containing these four lines.

```
16.0     3.0     2.0    13.0
 5.0    10.0    11.0     8.0
 9.0     6.0     7.0    12.0
 4.0    15.0    14.0     1.0
```

Store the file under the name `magik.dat`. Then the command

```
load magik.dat
```

reads the file and creates a variable, `magik`, containing our example matrix.

An easy way to read data into MATLAB in many text or binary formats is to use the Import Wizard.

M-Files

You can create your own matrices using *M-files*, which are text files containing MATLAB code. Use the MATLAB Editor or another text editor to create a file containing the same statements you would type at the MATLAB command line. Save the file under a name that ends in `.m`.

For example, create a file containing these five lines.

```
A = [ ...
16.0     3.0     2.0    13.0
 5.0    10.0    11.0     8.0
 9.0     6.0     7.0    12.0
 4.0    15.0    14.0     1.0 ];
```

Store the file under the name `magik.m`. Then the statement

```
magik
```

reads the file and creates a variable, `A`, containing our example matrix.

Concatenation

Concatenation is the process of joining small matrices to make bigger ones. In fact, you made your first matrix by concatenating its individual elements. The pair of square brackets, [], is the concatenation operator. For an example, start with the 4-by-4 magic square, A, and form

```
B = [A  A+32; A+48  A+16]
```

The result is an 8-by-8 matrix, obtained by joining the four submatrices.

```
B =

    16     3     2    13    48    35    34    45
     5    10    11     8    37    42    43    40
     9     6     7    12    41    38    39    44
     4    15    14     1    36    47    46    33
    64    51    50    61    32    19    18    29
    53    58    59    56    21    26    27    24
    57    54    55    60    25    22    23    28
    52    63    62    49    20    31    30    17
```

This matrix is half way to being another magic square. Its elements are a rearrangement of the integers 1:64. Its column sums are the correct value for an 8-by-8 magic square.

```
sum(B)

ans =
   260   260   260   260   260   260   260   260
```

But its row sums, sum(B')', are not all the same. Further manipulation is necessary to make this a valid 8-by-8 magic square.

Deleting Rows and Columns

You can delete rows and columns from a matrix using just a pair of square brackets. Start with

```
X = A;
```

Then, to delete the second column of X, use

```
X(:,2) = []
```

This changes X to

```
X =
    16     2    13
     5    11     8
     9     7    12
     4    14     1
```

If you delete a single element from a matrix, the result isn't a matrix anymore. So, expressions like

```
X(1,2) = []
```

result in an error. However, using a single subscript deletes a single element, or sequence of elements, and reshapes the remaining elements into a row vector. So

```
X(2:2:10) = []
```

results in

```
X =
    16     9     2     7    13    12     1
```

More About Matrices and Arrays

This sections shows you more about working with matrices and arrays, focusing on:

- Linear algebra
- Arrays
- Multivariate data

Linear Algebra

Informally, the terms *matrix* and *array* are often used interchangeably. More precisely, a *matrix* is a two-dimensional numeric array that represents a *linear transformation*. The mathematical operations defined on matrices are the subject of *linear algebra*.

Dürer's magic square

```
A =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    15    14     1
```

provides several examples that give a taste of MATLAB matrix operations. You've already seen the matrix transpose, A'. Adding a matrix to its transpose produces a *symmetric* matrix.

```
A + A'

ans =
    32     8    11    17
     8    20    17    23
    11    17    14    26
    17    23    26     2
```

The multiplication symbol, *, denotes the *matrix* multiplication involving inner products between rows and columns. Multiplying the transpose of a matrix by the original matrix also produces a symmetric matrix.

```
A'*A

ans =
   378   212   206   360
   212   370   368   206
   206   368   370   212
   360   206   212   378
```

The determinant of this particular matrix happens to be zero, indicating that the matrix is *singular*.

```
d = det(A)

d =
     0
```

The reduced row echelon form of A is not the identity.

```
R = rref(A)

R =
     1     0     0     1
     0     1     0    -3
     0     0     1     3
     0     0     0     0
```

Since the matrix is singular, it does not have an inverse. If you try to compute the inverse with

```
X = inv(A)
```

you will get a warning message

```
Warning: Matrix is close to singular or badly scaled.
         Results may be inaccurate. RCOND = 1.175530e-017.
```

Roundoff error has prevented the matrix inversion algorithm from detecting exact singularity. But the value of rcond, which stands for *reciprocal condition estimate*, is on the order of eps, the floating-point relative precision, so the computed inverse is unlikely to be of much use.

The eigenvalues of the magic square are interesting.

```
e = eig(A)

e =
   34.0000
    8.0000
    0.0000
   -8.0000
```

One of the eigenvalues is zero, which is another consequence of singularity. The largest eigenvalue is 34, the magic sum. That's because the vector of all ones is an eigenvector.

```
v = ones(4,1)

v =
     1
     1
     1
     1

A*v

ans =
    34
    34
    34
    34
```

When a magic square is scaled by its magic sum,

```
P = A/34
```

the result is a *doubly stochastic* matrix whose row and column sums are all one.

```
P =
    0.4706    0.0882    0.0588    0.3824
    0.1471    0.2941    0.3235    0.2353
    0.2647    0.1765    0.2059    0.3529
    0.1176    0.4412    0.4118    0.0294
```

Such matrices represent the transition probabilities in a *Markov process*. Repeated powers of the matrix represent repeated steps of the process. For our example, the fifth power

```
P^5
```

is

```
0.2507    0.2495    0.2494    0.2504
0.2497    0.2501    0.2502    0.2500
0.2500    0.2498    0.2499    0.2503
0.2496    0.2506    0.2505    0.2493
```

This shows that as k approaches infinity, all the elements in the kth power, p^k, approach $1/4$.

Finally, the coefficients in the characteristic polynomial

```
poly(A)
```

are

```
1   -34   -64   2176     0
```

This indicates that the characteristic polynomial

$det(A - \lambda I)$

is

$\lambda^4 - 34\lambda^3 - 64\lambda^2 + 2176\lambda$

The constant term is zero, because the matrix is singular, and the coefficient of the cubic term is -34, because the matrix is magic!

For More Information All of the MATLAB math functions are described in the “MATLAB Function Reference,” which is accessible from Help.

Arrays

When they are taken away from the world of linear algebra, matrices become two dimensional numeric arrays. Arithmetic operations on arrays are done element-by-element. This means that addition and subtraction are the same for arrays and matrices, but that multiplicative operations are different. MATLAB uses a dot, or decimal point, as part of the notation for multiplicative array operations.

The list of operators includes:

`+`	Addition
`-`	Subtraction
`.*`	Element-by-element multiplication
`./`	Element-by-element division
`.\`	Element-by-element left division
`.^`	Element-by-element power
`.'`	Unconjugated array transpose

If the Dürer magic square is multiplied by itself with array multiplication

```
A.*A
```

the result is an array containing the squares of the integers from 1 to 16, in an unusual order.

```
ans =
   256     9     4   169
    25   100   121    64
    81    36    49   144
    16   225   196     1
```

Building Tables

Array operations are useful for building tables. Suppose n is the column vector

```
n = (0:9)';
```

Then

```
pows = [n  n.^2  2.^n]
```

builds a table of squares and powers of two.

```
pows =
     0     0     1
     1     1     2
     2     4     4
     3     9     8
     4    16    16
     5    25    32
     6    36    64
     7    49   128
     8    64   256
     9    81   512
```

The elementary math functions operate on arrays element by element. So

```
format short g
x = (1:0.1:2)';
logs = [x log10(x)]
```

builds a table of logarithms.

```
logs =
     1.0           0
     1.1     0.04139
     1.2     0.07918
     1.3     0.11394
     1.4     0.14613
     1.5     0.17609
     1.6     0.20412
     1.7     0.23045
     1.8     0.25527
     1.9     0.27875
     2.0     0.30103
```

Multivariate Data

MATLAB uses column-oriented analysis for multivariate statistical data. Each column in a data set represents a variable and each row an observation. The `(i,j)`th element is the `i`th observation of the `j`th variable.

As an example, consider a data set with three variables:

- Heart rate
- Weight
- Hours of exercise per week

For five observations, the resulting array might look like

```
D =
       72          134          3.2
       81          201          3.5
       69          156          7.1
       82          148          2.4
       75          170          1.2
```

The first row contains the heart rate, weight, and exercise hours for patient 1, the second row contains the data for patient 2, and so on. Now you can apply many of MATLAB's data analysis functions to this data set. For example, to obtain the mean and standard deviation of each column:

```
mu = mean(D), sigma = std(D)

mu =
      75.8       161.8         3.48

sigma =
    5.6303      25.499       2.2107
```

For a list of the data analysis functions available in MATLAB, type

```
help datafun
```

If you have access to the Statistics Toolbox, type

```
help stats
```

Scalar Expansion

Matrices and scalars can be combined in several different ways. For example, a scalar is subtracted from a matrix by subtracting it from each element. The average value of the elements in our magic square is 8.5, so

```
B = A - 8.5
```

forms a matrix whose column sums are zero.

```
B =
      7.5     -5.5     -6.5      4.5
     -3.5      1.5      2.5     -0.5
      0.5     -2.5     -1.5      3.5
     -4.5      6.5      5.5     -7.5

sum(B)

ans =
     0     0     0     0
```

With scalar expansion, MATLAB assigns a specified scalar to all indices in a range. For example,

```
B(1:2,2:3) = 0
```

zeros out a portion of B

```
B =
      7.5        0        0      4.5
     -3.5        0        0     -0.5
      0.5     -2.5     -1.5      3.5
     -4.5      6.5      5.5     -7.5
```

Logical Subscripting

The logical vectors created from logical and relational operations can be used to reference subarrays. Suppose X is an ordinary matrix and L is a matrix of the same size that is the result of some logical operation. Then X(L) specifies the elements of X where the elements of L are nonzero.

This kind of subscripting can be done in one step by specifying the logical operation as the subscripting expression. Suppose you have the following set of data.

```
x =
  2.1 1.7 1.6 1.5 NaN 1.9 1.8 1.5 5.1 1.8 1.4 2.2 1.6 1.8
```

The NaN is a marker for a missing observation, such as a failure to respond to an item on a questionnaire. To remove the missing data with logical indexing, use `finite(x)`, which is true for all finite numerical values and false for `NaN` and `Inf`.

```
x = x(finite(x))
x =
  2.1 1.7 1.6 1.5 1.9 1.8 1.5 5.1 1.8 1.4 2.2 1.6 1.8
```

Now there is one observation, `5.1`, which seems to be very different from the others. It is an *outlier*. The following statement removes outliers, in this case those elements more than three standard deviations from the mean.

```
x = x(abs(x-mean(x)) <= 3*std(x))
x =
  2.1 1.7 1.6 1.5 1.9 1.8 1.5 1.8 1.4 2.2 1.6 1.8
```

For another example, highlight the location of the prime numbers in Dürer's magic square by using logical indexing and scalar expansion to set the nonprimes to 0.

```
A(~isprime(A)) = 0

A =
     0     3     2    13
     5     0    11     0
     0     0     7     0
     0     0     0     0
```

The find Function

The `find` function determines the indices of array elements that meet a given logical condition. In its simplest form, `find` returns a column vector of indices. Transpose that vector to obtain a row vector of indices. For example,

```
k = find(isprime(A))'
```

picks out the locations, using one-dimensional indexing, of the primes in the magic square.

```
k =
     2     5     9    10    11    13
```

Display those primes, as a row vector in the order determined by `k`, with

```
A(k)

ans =
     5     3     2    11     7    13
```

When you use `k` as a left-hand-side index in an assignment statement, the matrix structure is preserved.

```
A(k) = NaN

A =
    16   NaN   NaN   NaN
   NaN    10   NaN     8
     9     6   NaN    12
     4    15    14     1
```

Controlling Command Window Input and Output

So far, you have been using the MATLAB command line, typing commands and expressions, and seeing the results printed in the Command Window. This section describes how to:

- Control the appearance of the output values
- Suppress output from MATLAB commands
- Enter long commands at the command line
- Edit the command line

The format Command

The `format` command controls the numeric format of the values displayed by MATLAB. The command affects only how numbers are displayed, not how MATLAB computes or saves them. Here are the different formats, together with the resulting output produced from a vector `x` with components of different magnitudes.

Note To ensure proper spacing, use a fixed-width font, such as Fixedsys or Courier.

```
x = [4/3 1.2345e-6]

format short

   1.3333    0.0000

format short e

   1.3333e+000  1.2345e-006

format short g

   1.3333  1.2345e-006
```

```
format long

   1.33333333333333   0.00000123450000

format long e

   1.333333333333333e+000    1.234500000000000e-006

format long g

   1.33333333333333               1.2345e-006

format bank

   1.33          0.00

format rat

   4/3          1/810045

format hex

   3ff5555555555555   3eb4b6231abfd271
```

If the largest element of a matrix is larger than 10^3 or smaller than 10^{-3}, MATLAB applies a common scale factor for the short and long formats.

In addition to the `format` commands shown above

```
format compact
```

suppresses many of the blank lines that appear in the output. This lets you view more information on a screen or window. If you want more control over the output format, use the `sprintf` and `fprintf` functions.

Suppressing Output

If you simply type a statement and press **Return** or **Enter**, MATLAB automatically displays the results on screen. However, if you end the line with a semicolon, MATLAB performs the computation but does not display any output. This is particularly useful when you generate large matrices. For example,

```
A = magic(100);
```

Entering Long Command Lines

If a statement does not fit on one line, use three periods, `...`, followed by **Return** or **Enter** to indicate that the statement continues on the next line. For example,

```
s = 1 -1/2 + 1/3 -1/4 + 1/5 - 1/6 + 1/7 ...
      - 1/8 + 1/9 - 1/10 + 1/11 - 1/12;
```

Blank spaces around the `=`, `+`, and `-` signs are optional, but they improve readability.

Command Line Editing

Various arrow and control keys on your keyboard allow you to recall, edit, and reuse commands you have typed earlier. For example, suppose you mistakenly enter

```
rho = (1 + sqt(5))/2
```

You have misspelled `sqrt`. MATLAB responds with

```
Undefined function or variable 'sqt'.
```

Instead of retyping the entire line, simply press the ↑ key. The misspelled command is redisplayed. Use the ← key to move the cursor over and insert the missing `r`. Repeated use of the ↑ key recalls earlier lines. Typing a few characters and then the ↑ key finds a previous line that begins with those characters. You can also copy previously executed commands from the Command History. For more information, see “Command History” on page 3-7.

The list of available command line editing keys is different on different computers. Experiment to see which of the following keys is available on your machine. (Many of these keys will be familiar to users of the Emacs editor.)

↑	**Ctrl+p**	Recall previous line
↓	**Ctrl+n**	Recall next line
←	**Ctrl+b**	Move back one character
→	**Ctrl+f**	Move forward one character
Ctrl+→	**Ctrl+r**	Move right one word
Ctrl+←	**Ctrl+l**	Move left one word
Home	**Ctrl+a**	Move to beginning of line
End	**Ctrl+e**	Move to end of line
Esc	**Ctrl+u**	Clear line
Del	**Ctrl+d**	Delete character at cursor
Backspace	**Ctrl+h**	Delete character before cursor
	Ctrl+k	Delete to end of line

Tab Completion

MATLAB completes the name of a function, variable, filename, or handle graphics property if you type the first few letters and then press the **Tab** key. If there is a unique name, the name is automatically completed. If there is more than one name that starts with the letters you typed, press the **Tab** key again to see a list of the possibilities.

5

Graphics

Basic Plotting

MATLAB has extensive facilities for displaying vectors and matrices as graphs, as well as annotating and printing these graphs. This section describes a few of the most important graphics functions and provides examples of some typical applications.

For More Information “Graphics” and “3-D Visualization” provide in-depth coverage of MATLAB graphics and visualization tools. Access these from Help.

Creating a Plot

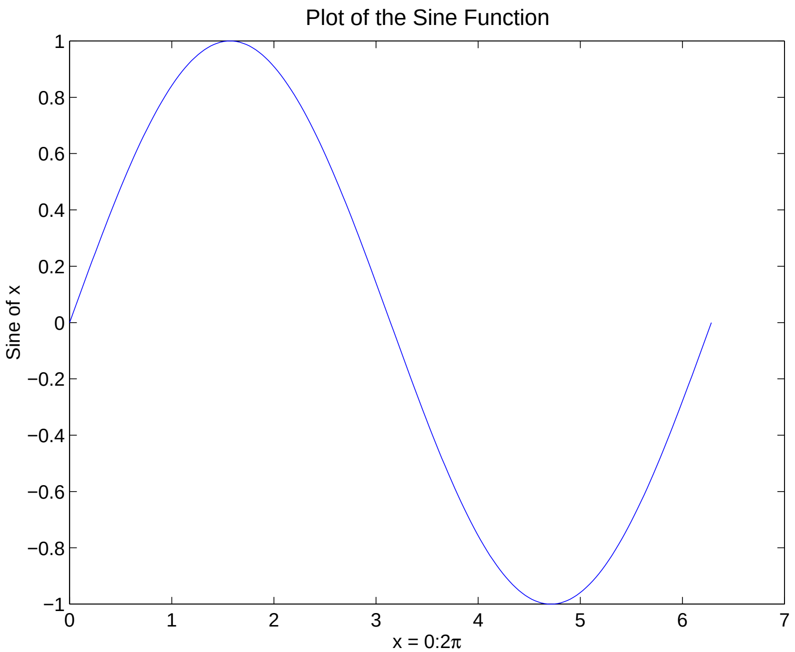
The `plot` function has different forms, depending on the input arguments. If `y` is a vector, `plot(y)` produces a piecewise linear graph of the elements of `y` versus the index of the elements of `y`. If you specify two vectors as arguments, `plot(x,y)` produces a graph of `y` versus `x`.

For example, these statements use the `colon` operator to create a vector of `x` values ranging from zero to 2π, compute the sine of these values, and plot the result.

```
x = 0:pi/100:2*pi;
y = sin(x);
plot(x,y)
```

Now label the axes and add a title. The characters `\pi` create the symbol π.

```
xlabel('x = 0:2\pi')
ylabel('Sine of x')
title('Plot of the Sine Function','FontSize',12)
```



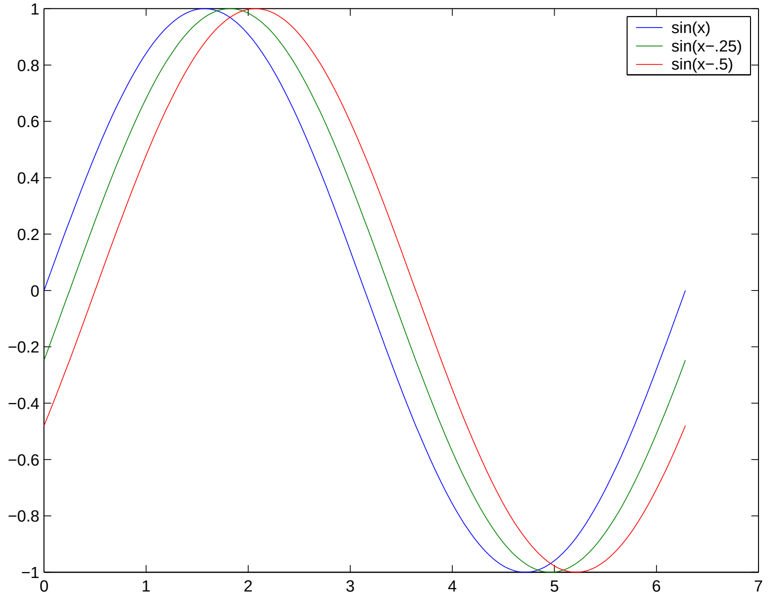
Multiple Data Sets in One Graph

Multiple x-y pair arguments create multiple graphs with a single call to plot. MATLAB automatically cycles through a predefined (but user settable) list of colors to allow discrimination between each set of data. For example, these statements plot three related functions of x, each curve in a separate distinguishing color.

```
y2 = sin(x-.25);
y3 = sin(x-.5);
plot(x,y,x,y2,x,y3)
```

The legend command provides an easy way to identify the individual plots.

```
legend('sin(x)','sin(x-.25)','sin(x-.5)')
```



For More Information See "Defining the Color of Lines for Plotting" in "Axes Properties" in Help.

Specifying Line Styles and Colors

It is possible to specify color, line styles, and markers (such as plus signs or circles) when you plot your data using the `plot` command.

```
plot(x,y,'color_style_marker')
```

color_style_marker is a string containing from one to four characters (enclosed in single quotation marks) constructed from a color, a line style, and a marker type:

- Color strings are `'c'`, `'m'`, `'y'`, `'r'`, `'g'`, `'b'`, `'w'`, and `'k'`. These correspond to cyan, magenta, yellow, red, green, blue, white, and black.

- Linestyle strings are '-' for solid, '--' for dashed, ':' for dotted, '-.' for dash-dot, and 'none' for no line.
- The marker types are '+', 'o', '*', and 'x' and the filled marker types 's' for square, 'd' for diamond, '^' for up triangle, 'v' for down triangle, '>' for right triangle, '<' for left triangle, 'p' for pentagram, 'h' for hexagram, and none for no marker.

You can also edit color, line style, and markers interactively. See "Editing Plots" on page 5-14 for more information.

Plotting Lines and Markers

If you specify a marker type but not a linestyle, MATLAB draws only the marker. For example,

```
plot(x,y,'ks')
```

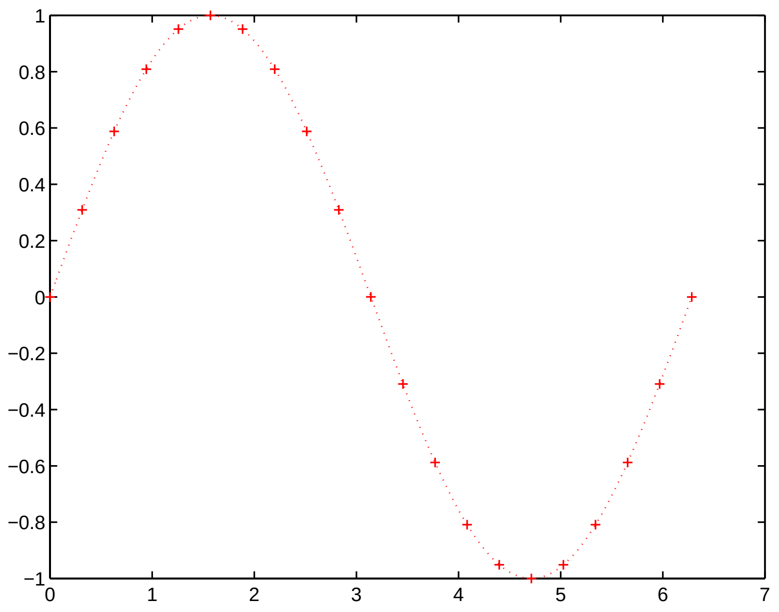
plots black squares at each data point, but does not connect the markers with a line.

The statement

```
plot(x,y,'r:+')
```

plots a red dotted line and places plus sign markers at each data point. You may want to use fewer data points to plot the markers than you use to plot the lines. This example plots the data twice using a different number of points for the dotted line and marker plots.

```
x1 = 0:pi/100:2*pi;
x2 = 0:pi/10:2*pi;
plot(x1,sin(x1),'r:',x2,sin(x2),'r+')
```



For More Information See "Basic Plotting" in Help for more examples of plotting options.

Imaginary and Complex Data

When the arguments to `plot` are complex, the imaginary part is ignored *except* when `plot` is given a single complex argument. For this special case, the command is a shortcut for a plot of the real part versus the imaginary part. Therefore,

```
plot(Z)
```

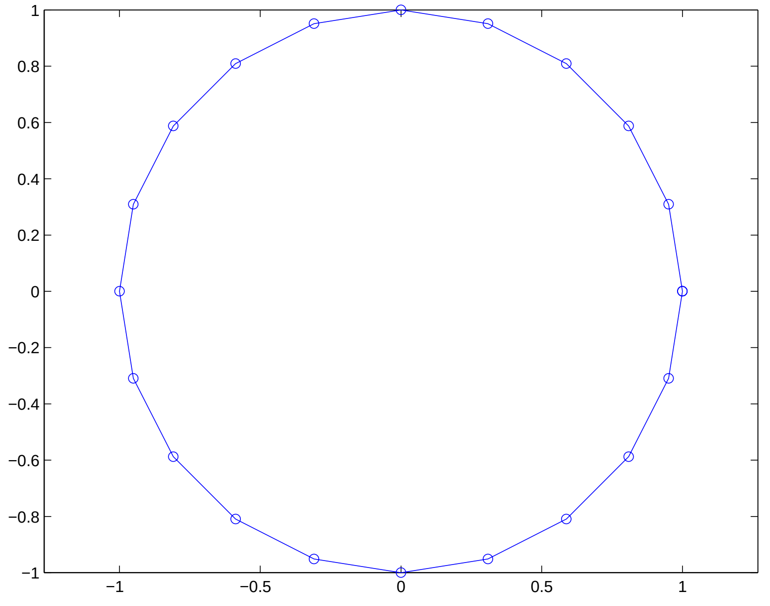
where `Z` is a complex vector or matrix, is equivalent to

```
plot(real(Z),imag(Z))
```

For example,

```
t = 0:pi/10:2*pi;
plot(exp(i*t),'-o')
axis equal
```

draws a 20-sided polygon with little circles at the vertices. The command, `axis equal`, makes the individual tick mark increments on the *x*- and *y*-axes the same length, which makes this plot more circular in appearance.



Adding Plots to an Existing Graph

The `hold` command enables you to add plots to an existing graph. When you type

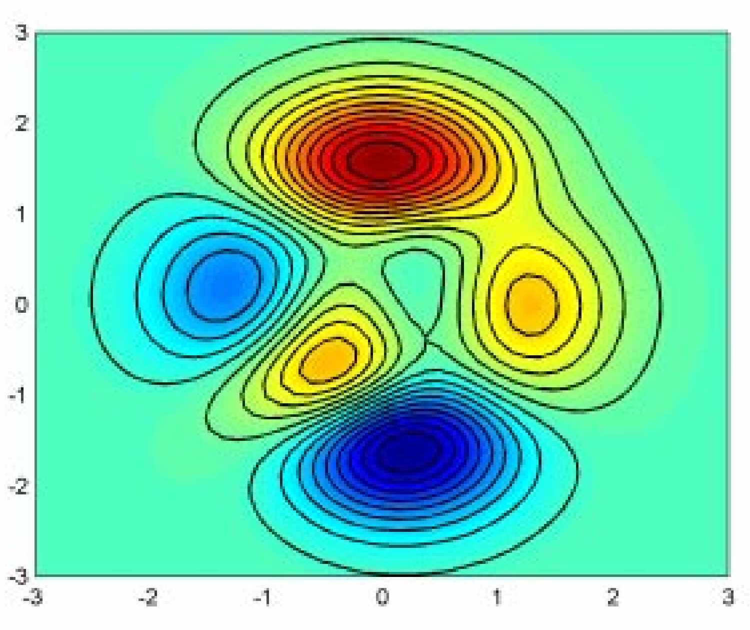
```
hold on
```

MATLAB does not replace the existing graph when you issue another plotting command; it adds the new data to the current graph, rescaling the axes if necessary.

For example, these statements first create a contour plot of the peaks function, then superimpose a pseudocolor plot of the same function.

```
[x,y,z] = peaks;
contour(x,y,z,20,'k')
hold on
pcolor(x,y,z)
shading interp
hold off
```

The `hold on` command causes the `pcolor` plot to be combined with the `contour` plot in one figure.



For More Information See "Creating Specialized Plots" in Help for information on a variety of graph types.

Figure Windows

Graphing functions automatically open a new figure window if there are no figure windows already on the screen. If a figure window exists, MATLAB uses that window for graphics output. If there are multiple figure windows open, MATLAB targets the one that is designated the "current figure" (the last figure used or clicked in).

To make an existing figure window the current figure, you can click the mouse while the pointer is in that window or you can type

```
figure(n)
```

where `n` is the number in the figure title bar. The results of subsequent graphics commands are displayed in this window.

To open a new figure window and make it the current figure, type

```
figure
```

Clearing the Figure for a New Plot

When a figure already exists, most plotting commands clear the axes and use this figure to create the new plot. However, these commands do not reset figure properties, such as the background color or the colormap. If you have set any figure properties in the previous plot, you may want to use the `clf` command with the `reset` option,

```
clf reset
```

before creating your new plot to set the figure's properties to their defaults.

For More Information See "Figure Properties" and the reference page for the `figure` command in Help. See "Controlling Graphics Output" for information on how to control property resetting in your graphics programs.

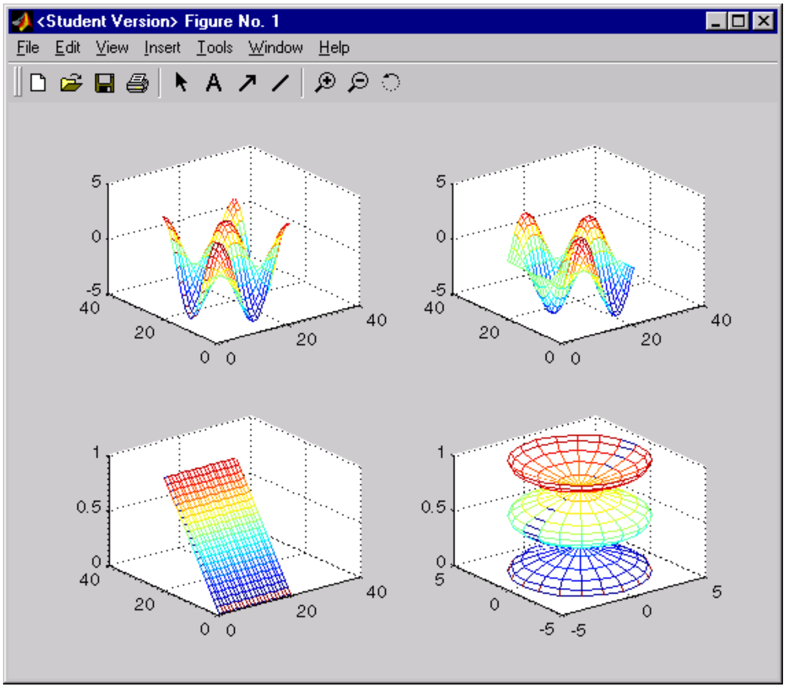
Multiple Plots in One Figure

The `subplot` command enables you to display multiple plots in the same window or print them on the same piece of paper. Typing

```
subplot(m,n,p)
```

partitions the figure window into an m-by-n matrix of small subplots and selects the pth subplot for the current plot. The plots are numbered along first the top row of the figure window, then the second row, and so on. For example, these statements plot data in four different subregions of the figure window.

```
t = 0:pi/10:2*pi;
[X,Y,Z] = cylinder(4*cos(t));
subplot(2,2,1); mesh(X)
subplot(2,2,2); mesh(Y)
subplot(2,2,3); mesh(Z)
subplot(2,2,4); mesh(X,Y,Z)
```



Controlling the Axes

The axis command supports a number of options for setting the scaling, orientation, and aspect ratio of plots. You can also set these options interactively. See "Editing Plots" on page 5-14 for more information.

Setting Axis Limits

By default, MATLAB finds the maxima and minima of the data to choose the axis limits to span this range. The `axis` command enables you to specify your own limits

```
axis([xmin xmax ymin ymax])
```

or for three-dimensional graphs,

```
axis([xmin xmax ymin ymax zmin zmax])
```

Use the command

```
axis auto
```

to re-enable MATLAB's automatic limit selection.

Setting Axis Aspect Ratio

`axis` also enables you to specify a number of predefined modes. For example,

```
axis square
```

makes the *x*-axes and *y*-axes the same length.

```
axis equal
```

makes the individual tick mark increments on the *x*- and *y*-axes the same length. This means

```
plot(exp(i*[0:pi/10:2*pi]))
```

followed by either `axis square` or `axis equal` turns the oval into a proper circle.

```
axis auto normal
```

returns the axis scaling to its default, automatic mode.

Setting Axis Visibility

You can use the `axis` command to make the axis visible or invisible.

```
axis on
```

makes the axis visible. This is the default.

```
axis off
```

makes the axis invisible.

Setting Grid Lines

The grid command toggles grid lines on and off. The statement

```
grid on
```

turns the grid lines on and

```
grid off
```

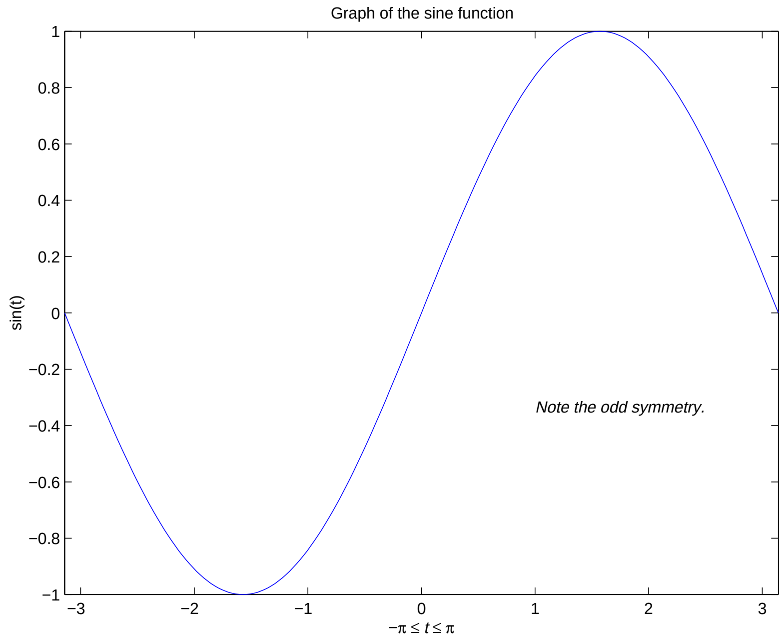
turns them back off again.

For More Information See the axis and axes reference pages and “Axes Properties” in Help.

Axis Labels and Titles

The xlabel, ylabel, and zlabel commands add *x*-, *y*-, and *z*-axis labels. The title command adds a title at the top of the figure and the text function inserts text anywhere in the figure. A subset of TeX notation produces Greek letters. You can also set these options interactively. See “Editing Plots” on page 5-14 for more information.

```
t = -pi:pi/100:pi;
y = sin(t);
plot(t,y)
axis([-pi pi -1 1])
xlabel('-\pi \leq {\itt} \leq \pi')
ylabel('sin(t)')
title('Graph of the sine function')
text(1,-1/3,'{\itNote the odd symmetry.}')
```



For More Information See "Formatting Graphs" in Help for additional information on adding labels and annotations to your graphs.

Saving a Figure

To save a figure, select **Save** from the **File** menu. The figure is saved as a FIG-file, which you can load using the `open` or `hgload` commands.

Formats for Importing into Other Applications

You can export the figure as a standard graphics format, such as TIFF, for use with other applications. To do this, select **Export** from the **File** menu. You can also export figures from the command line using the `saveas` and `print` commands.

Editing Plots

MATLAB formats a graph to provide readability, setting the scale of axes, including tick marks on the axes, and using color and line style to distinguish the plots in the graph. However, if you are creating presentation graphics, you may want to change this default formatting or add descriptive labels, titles, legends and other annotations to help explain your data.

MATLAB supports two ways to edit the plots you create:

- Using the mouse to select and edit objects interactively
- Using MATLAB functions at the command-line or in an M-file

Interactive Plot Editing

If you enable plot editing mode in the MATLAB figure window, you can perform point-and-click editing of the objects in your graph. In this mode, you select the object or objects you want to edit by double-clicking on it. This starts the Property Editor, which provides access to properties of the object that control its appearance and behavior.

For more information about interactive editing, see "Using Plot Editing Mode" on page 5-15. For information about editing object properties in plot editing mode, see "Using the Property Editor" on page 5-16.

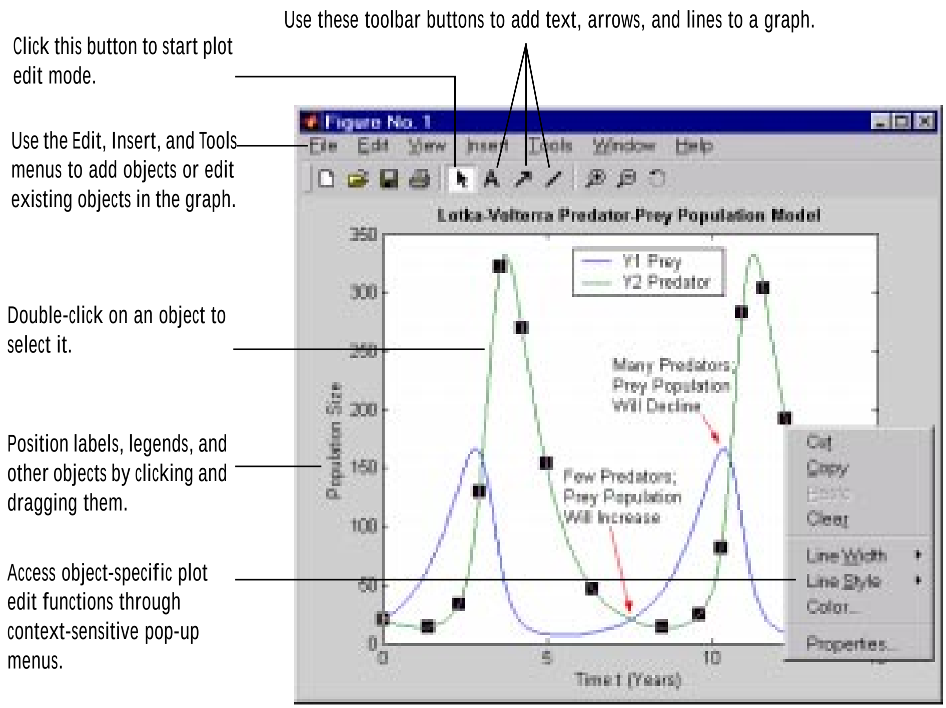
Note Plot editing mode provides an alternative way to access the properties of MATLAB graphic objects. However, you can only access a subset of object properties through this mechanism. You may need to use a combination of interactive editing and command line editing to achieve the effect you desire.

Using Functions to Edit Graphs

If you prefer to work from the MATLAB command line or if you are creating an M-file, you can use MATLAB commands to edit the graphs you create. Taking advantage of MATLAB's Handle Graphics system, you can use the `set` and `get` commands to change the properties of the objects in a graph. For more information about using command line, see "Handle Graphics" on page 5-28.

Using Plot Editing Mode

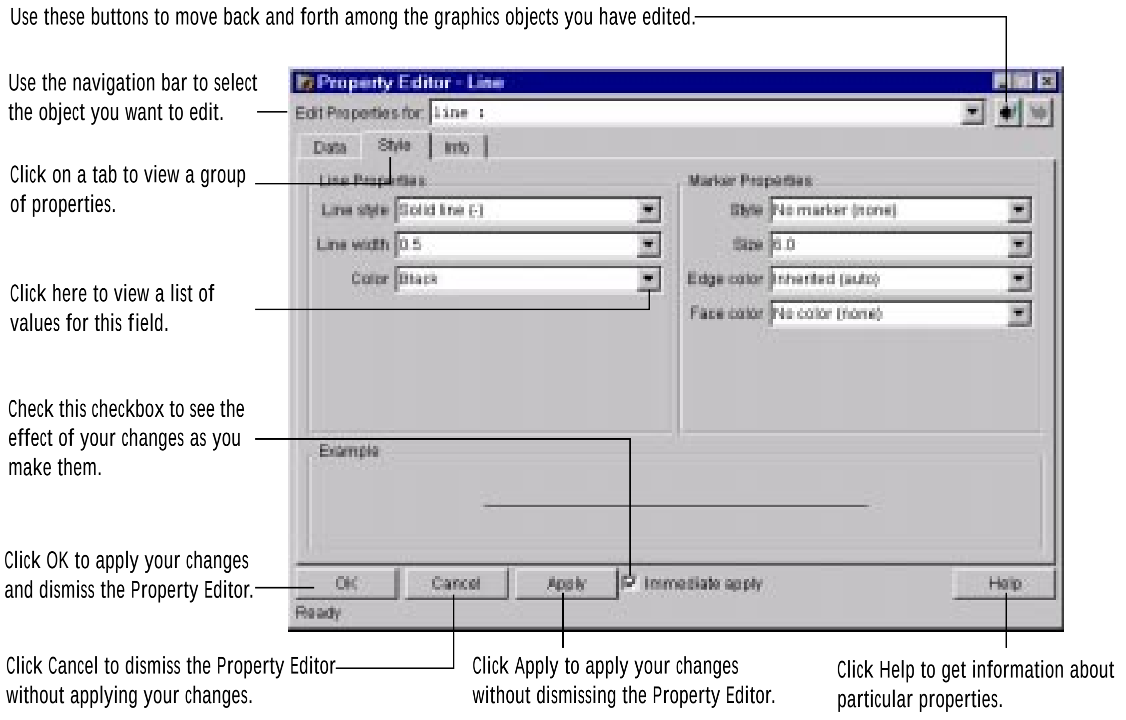
The MATLAB figure window supports a point-and-click style editing mode that you can use to customize the appearance of your graph. The following illustration shows a figure window with plot editing mode enabled and labels the main plot editing mode features.



Using the Property Editor

In plot editing mode, you can use a graphical user interface, called the Property Editor, to edit the properties of objects in the graph. The Property Editor provides access to many properties of the root, figure, axes, line, light, patch, image, surfaces rectangle, and text objects. For example, using the Property Editor, you can change the thickness of a line, add titles and axes labels, add lights, and perform many other plot editing tasks.

This figure shows the components of the Property Editor interface.



Starting the Property Editor

You start the Property Editor by double-clicking on an object in a graph, such as a line, or by right-clicking on an object and selecting the **Properties** option from the object's context menu.

You can also start the Property Editor by selecting either the **Figure Properties**, **Axes Properties**, or **Current Object Properties** from the figure window **Edit** menu. These options automatically enable plot editing mode, if it is not already enabled.

Once you start the Property Editor, keep it open throughout an editing session. It provides access to all the objects in the graph. If you click on another object in the graph, the Property Editor displays the set of panels associated with that object type. You can also use the Property Editor's navigation bar to select an object in the graph to edit.

To save a figure, select **Save** from the **File** menu. To save it using a graphics format, such as TIFF, for use with other applications, select **Export** from the **File** menu. You can also save from the command line – use the saveas command, including any options to save the figure in a different format.

Mesh and Surface Plots

MATLAB defines a surface by the *z*-coordinates of points above a grid in the *x*-*y* plane, using straight lines to connect adjacent points. The `mesh` and `surf` plotting functions display surfaces in three dimensions. `mesh` produces wireframe surfaces that color only the lines connecting the defining points. `surf` displays both the connecting lines and the faces of the surface in color.

Visualizing Functions of Two Variables

To display a function of two variables, $z = f(x,y)$:

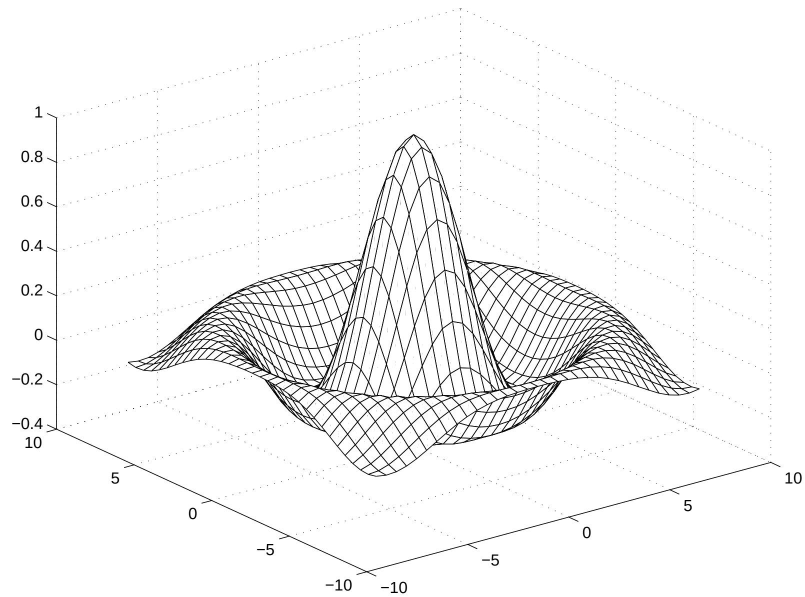
- Generate `X` and `Y` matrices consisting of repeated rows and columns, respectively, over the domain of the function.
- Use `X` and `Y` to evaluate and graph the function.

The `meshgrid` function transforms the domain specified by a single vector or two vectors `x` and `y` into matrices `X` and `Y` for use in evaluating functions of two variables. The rows of `X` are copies of the vector `x` and the columns of `Y` are copies of the vector `y`.

Example – Graphing the sinc Function

This example evaluates and graphs the two-dimensional *sinc* function, sin(*r*)/*r*, between the *x* and *y* directions. `R` is the distance from origin, which is at the center of the matrix. Adding `eps` (a MATLAB command that returns the smallest floating-point number on your system) avoids the indeterminate 0/0 at the origin.

```
[X,Y] = meshgrid(-8:.5:8);
R = sqrt(X.^2 + Y.^2) + eps;
Z = sin(R)./R;
mesh(X,Y,Z,'EdgeColor','black')
```



By default, MATLAB colors the mesh using the current colormap. However, this example uses a single-colored mesh by specifying the `EdgeColor` surface property. See the `surface` reference page for a list of all surface properties.

You can create a transparent mesh by disabling hidden line removal.

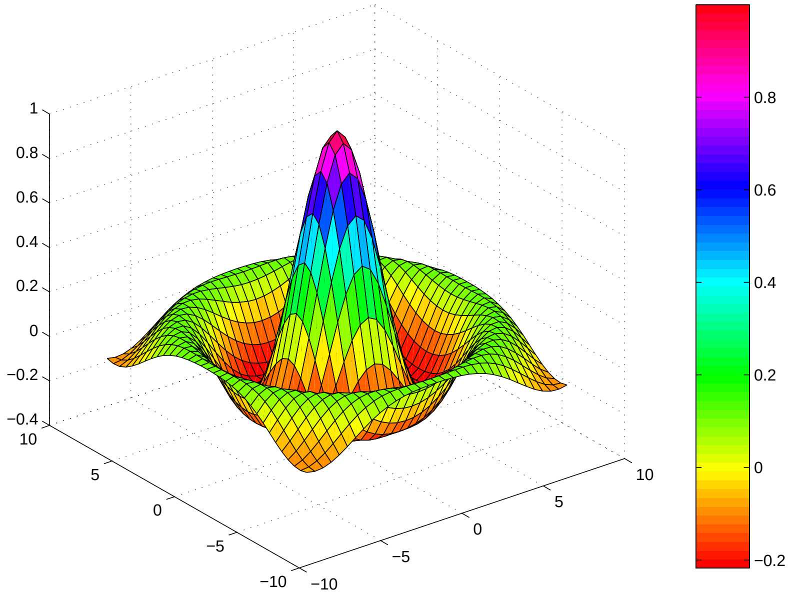
```
hidden off
```

See the `hidden` reference page for more information on this option.

Example – Colored Surface Plots

A surface plot is similar to a mesh plot except the rectangular faces of the surface are colored. The color of the faces is determined by the values of `Z` and the colormap (a `colormap` is an ordered list of colors). These statements graph the *sinc* function as a surface plot, select a colormap, and add a color bar to show the mapping of data to color.

```
surf(X,Y,Z)
colormap hsv
colorbar
```



See the `colormap` reference page for information on colormaps.

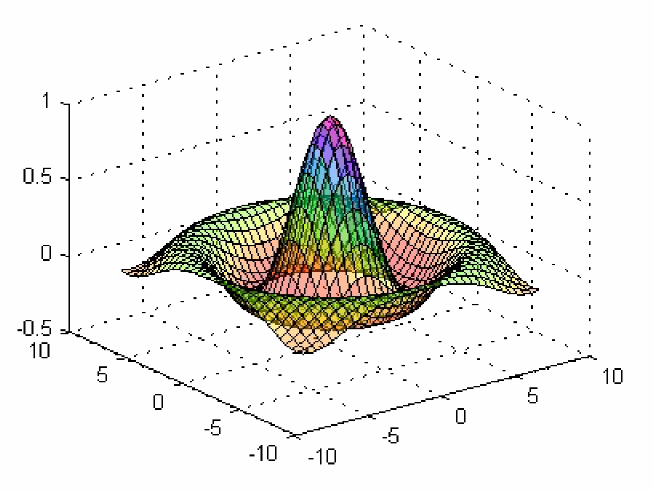
For More Information See “Creating 3-D Graphs” in Help for more information on surface plots.

Transparent Surfaces

You can make the faces of a surface transparent to a varying degree. Transparency (referred to as the alpha value) can be specified for the whole object or can be based on an alphamap, which behaves in a way analogous to colormaps. For example,

```
surf(X,Y,Z)
colormap hsv
alpha(.4)
```

produces a surface with a face alpha value of 0.4. Alpha values range from 0 (completely transparent) to 1 (not transparent).



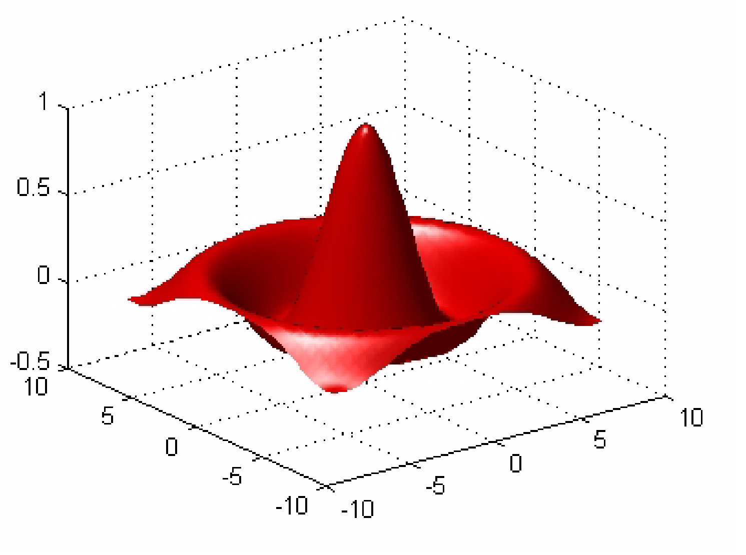
For More Information See "Transparency" in Help for more information on using this feature.

Surface Plots with Lighting

Lighting is the technique of illuminating an object with a directional light source. In certain cases, this technique can make subtle differences in surface shape easier to see. Lighting can also be used to add realism to three-dimensional graphs.

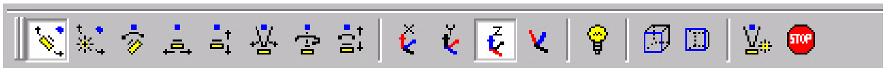
This example uses the same surface as the previous examples, but colors it red and removes the mesh lines. A light object is then added to the left of the "camera" (that is the location in space from where you are viewing the surface).

```
surf(X,Y,Z,'FaceColor','red','EdgeColor','none')
camlight left; lighting phong
```

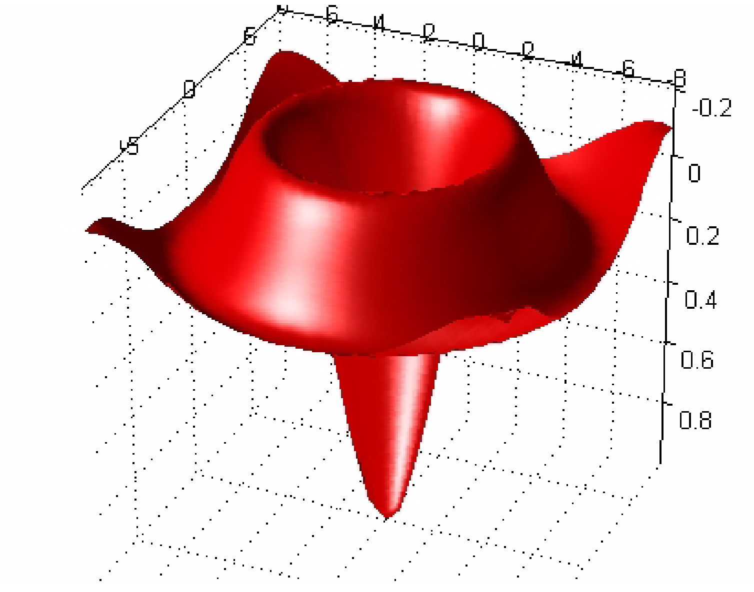


Manipulating the Surface

The Camera Toolbar provides a way to interactively explore 3-D graphics. Display the toolbar by selecting **Camera Toolbar** from the figure window's **View** menu. Here is the toolbar with the orbit camera tool selected:



The Camera Toolbar enables you to move the camera around the surface object, zoom, add a light, and perform other viewing operations without issuing commands. The following picture shows the surface viewed by orbiting the camera toward the bottom. A scene light has been added to illuminate the underside of the surface, which is not lit by the light added in the previous section.



For More Information See the "Lighting as a Visualization Tool" and "View Control with the Camera Toolbar" in Help for information on these techniques.

Images

Two-dimensional arrays can be displayed as *images*, where the array elements determine brightness or color of the images. For example, the statements

```
load durer
whos
Name          Size         Bytes  Class

  X           648x509      2638656  double array
  caption     2x28             112  char array
  map         128x3           3072  double array
```

load the file `durer.mat`, adding three variables to the workspace. The matrix `X` is a 648-by-509 matrix and `map` is a 128-by-3 matrix that is the colormap for this image.

Note MAT-files, such as `durer.mat`, are binary files that can be created on one platform and later read by MATLAB on a different platform.

The elements of `X` are integers between 1 and 128, which serve as indices into the colormap, `map`. Then

```
image(X)
colormap(map)
axis image
```

reproduces Dürer's etching shown at the beginning of this book. A high resolution scan of the magic square in the upper right corner is available in another file. Type

```
load detail
```

and then use the uparrow key on your keyboard to reexecute the `image`, `colormap`, and `axis` commands. The statement

```
colormap(hot)
```

adds some unusual coloring to the sixteenth century etching. The function `hot` generates a colormap containing shades of reds, oranges, and yellows.

Typically a given image matrix has a specific colormap associated with it. See the `colormap` reference page for a list of other predefined colormaps.

For More Information See “Displaying Bit-Mapped Images” in Help for information on the image processing capabilities of MATLAB.

Printing Graphics

You can print a MATLAB figure directly on a printer connected to your computer or you can export the figure to one of the standard graphic file formats supported by MATLAB. There are two ways to print and export figures:

- Using the **Print** option under the **File** menu
- Using the `print` command

Printing from the Menu

There are four menu options under the **File** menu that pertain to printing:

- The **Page Setup** option displays a dialog box that enables you to adjust characteristics of the figure on the printed page.
- The **Print Setup** option displays a dialog box that sets printing defaults, but does not actually print the figure.
- The **Print Preview** option enables you to view the figure the way it will look on the printed page.
- The **Print** option displays a dialog box that lets you select standard printing options and print the figure.

Generally, use **Print Preview** to determine whether the printed output is what you want. If not, use the **Page Setup** dialog box to change the output settings. Select the **Page Setup** dialog box **Help** button to display information on how to set up the page.

Exporting Figure to Graphics Files

The **Export** option under the **File** menu enables you to export the figure to a variety of standard graphics file formats.

Using the Print Command

The `print` command provides more flexibility in the type of output sent to the printer and allows you to control printing from M-files. The result can be sent directly to your default printer or stored in a specified file. A wide variety of output formats, including TIFF, JPEG, and PostScript, is available.

For example, this statement saves the contents of the current figure window as color Encapsulated Level 2 PostScript in the file called `magicsquare.eps`. It also includes a TIFF preview, which enables most word processors to display the picture

```
print -depsc2 -tiff magicsquare.eps
```

To save the same figure as a TIFF file with a resolution of 200 dpi, use the command

```
print -dtiff -r200 magicsquare.tiff
```

If you type `print` on the command line,

```
print
```

MATLAB prints the current figure on your default printer.

For More Information See the `print` command reference page and "Basic Printing and Exporting" in Help for more information on printing.

Handle Graphics

When you use a plotting command, MATLAB creates the graph using various graphics objects, such as lines, text, and surfaces (see “Graphics Objects” on page 5-28 for a complete list). All graphics objects have properties that control the appearance and behavior of the object. MATLAB enables you to query the value of each property and set the value of most properties.

Whenever MATLAB creates a graphics object, it assigns an identifier (called a handle) to the object. You can use this handle to access the object’s properties. Handle Graphics is useful if you want to:

- Modify the appearance of graphs.
- Create custom plotting commands by writing M-files that create and manipulate objects directly.

Graphics Objects

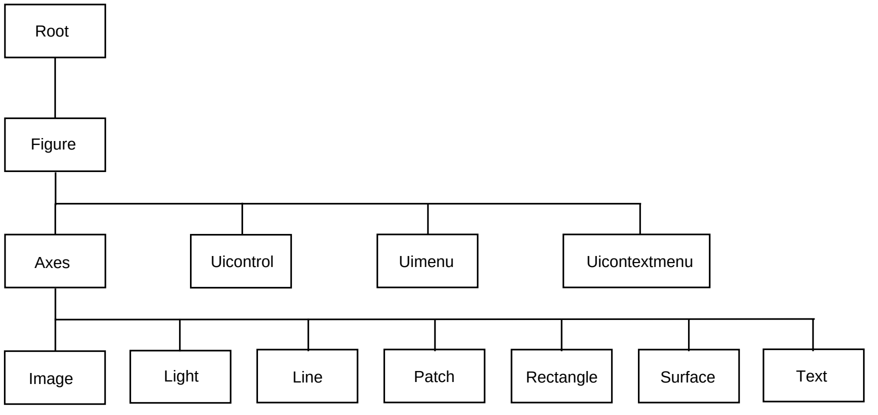
Graphics objects are the basic elements used to display graphics and user interface elements. This table lists the graphics objects.

Object	Description
Root	Top of the hierarchy corresponding to the computer screen
Figure	Window used to display graphics and user interfaces
Axes	Axes for displaying graphs in a figure
Uicontrol	User interface control that executes a function in response to user interaction
Uimenu	User-defined figure window menu
Uicontextmenu	Pop-up menu invoked by right clicking on a graphics object
Image	Two-dimensional pixel-based picture

Object	Description
Light	Light sources that affect the coloring of patch and surface objects
Line	Line used by functions such as `plot`, `plot3`, `semilogx`
Patch	Filled polygon with edges
Rectangle	Two-dimensional shape varying from rectangles to ovals
Surface	Three-dimensional representation of matrix data created by plotting the value of the data as heights above the *x-y* plane
Text	Character string

Object Hierarchy

The objects are organized in a tree structured hierarchy reflecting their interdependence. For example, line objects require axes objects as a frame of reference. In turn, axes objects exist only within figure objects. This diagram illustrates the tree structure.



Creating Objects

Each object has an associated function that creates the object. These functions have the same name as the objects they create. For example, the `text` function creates text objects, the `figure` function creates figure objects, and so on. MATLAB's high-level graphics functions (like `plot` and `surf`) call the appropriate low-level function to draw their respective graphics. For more information about an object and a description of its properties, see the reference page for the object's creation function. Object creation functions have the same name as the object. For example, the object creation function for axes objects is called `axes`.

Commands for Working with Objects

This table lists commands commonly used when working with objects.

Function	Purpose
`copyobj`	Copy graphics object
`delete`	Delete an object
`findobj`	Find the handle of objects having specified property values
`gca`	Return the handle of the current axes
`gcf`	Return the handle of the current figure
`gco`	Return the handle of the current object
`get`	Query the value of an objects properties
`set`	Set the value of an objects properties

For More Information See the "MATLAB Function Reference" in Help for a description of each of these functions.

Setting Object Properties

All object properties have default values. However, you may find it useful to change the settings of some properties to customize your graph. There are two ways to set object properties:

- Specify values for properties when you create the object.
- Set the property value on an object that already exists.

For More Information See "Handle Graphics Objects" in Help for information on graphics objects.

Setting Properties from Plotting Commands

You can specify object property values as arguments to object creation functions as well as with plotting function, such as `plot`, `mesh`, and `surf`.

For example, plotting commands that create lines or surfaces enable you to specify property name/property value pairs as arguments. The command

```
plot(x,y,'LineWidth',1.5)
```

plots the data in the variables `x` and `y` using lines having a `LineWidth` property set to 1.5 points (one point = 1/72 inch). You can set any line object property this way.

Setting Properties of Existing Objects

To modify the property values of existing objects, you can use the `set` command or, if plot editing mode is enabled, the Property Editor. The Property Editor provides a graphical user interface to many object properties. This section describes how to use the set command. See "Using the Property Editor" on page 5-16 for more information.

Many plotting commands can return the handles of the objects created so you can modify the objects using the `set` command. For example, these statements plot a five-by-five matrix (creating five lines, one per column) and then set the `Marker` to a square and the `MarkerFaceColor` to green.

```
h = plot(magic(5));
set(h,'Marker','s',MarkerFaceColor','g')
```

In this case, h is a vector containing five handles, one for each of the five lines in the plot. The set statement sets the Marker and MarkerFaceColor properties of all lines to the same values.

Setting Multiple Property Values

If you want to set the properties of each line to a different value, you can use cell arrays to store all the data and pass it to the set command. For example, create a plot and save the line handles.

```
h = plot(magic(5));
```

Suppose you want to add different markers to each line and color the marker's face color to the same color as the line. You need to define two cell arrays – one containing the property names and the other containing the desired values of the properties.

The prop_name cell array contains two elements.

```
prop_name(1) = {'Marker'};
prop_name(2) = {'MarkerFaceColor'};
```

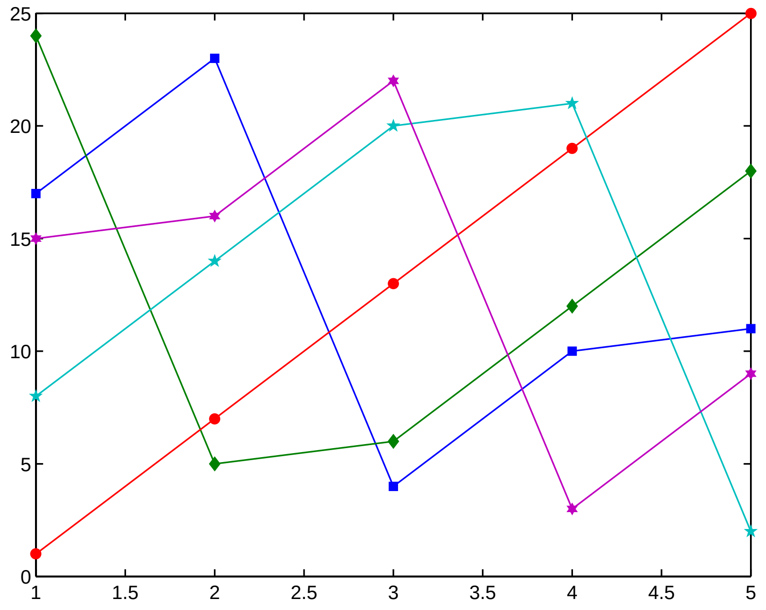
The prop_values cell array contains 10 values – five values for the Marker property and five values for the MarkerFaceColor property. Notice that prop_values is a two-dimensional cell array. The first dimension indicates which handle in h the values apply to and the second dimension indicates which property the value is assigned to.

```
prop_values(1,1) = {'s'};
prop_values(1,2) = {get(h(1),'Color')};
prop_values(2,1) = {'d'};
prop_values(2,2) = {get(h(2),'Color')};
prop_values(3,1) = {'o'};
prop_values(3,2) = {get(h(3),'Color')};
prop_values(4,1) = {'p'};
prop_values(4,2) = {get(h(4),'Color')};
prop_values(5,1) = {'h'};
prop_values(5,2) = {get(h(5),'Color')};
```

The MarkerFaceColor is always assigned the value of the corresponding line's color (obtained by getting the line's Color property with the get command).

After defining the cell arrays, call set to specify the new property values.

```
set(h,prop_name,prop_values)
```



For More Information See “Structures and Cell Arrays” in Help for information on cell arrays.

Finding the Handles of Existing Objects

The `findobj` command enables you to obtain the handles of graphics objects by searching for objects with particular property values. With `findobj` you can specify the value of any combination of properties, which makes it easy to pick one object out of many. For example, you may want to find the blue line with square marker having blue face color.

You can also specify which figures or axes to search, if there is more than one. The following sections provide examples illustrating how to use `findobj`.

Finding All Objects of a Certain Type

Since all objects have a `Type` property that identifies the type of object, you can find the handles of all occurrences of a particular type of object. For example,

```
h = findobj('Type','line');
```

finds the handles of all line objects.

Finding Objects with a Particular Property

You can specify multiple properties to narrow the search. For example,

```
h = findobj('Type','line','Color','r','LineStyle',':');
```

finds the handles of all red, dotted lines.

Limiting the Scope of the Search

You can specify the starting point in the object hierarchy by passing the handle of the starting figure or axes as the first argument. For example,

```
h = findobj(gca,'Type','text','String','\pi/2');
```

finds the string $\pi/2$ only within the current axes.

Using findobj as an Argument

Since `findobj` returns the handles it finds, you can use it in place of the handle argument. For example,

```
set(findobj('Type','line','Color','red'),'LineStyle',':')
```

finds all red lines and sets their line style to dotted.

For More Information See "Accessing Object Handles" in Help for more information.

Graphics User Interfaces

Here is a simple example illustrating how to use Handle Graphics to build user interfaces. The statement

```
b = uicontrol('Style','pushbutton', ...
      'Units','normalized', ...
      'Position',[.5 .5 .2 .1], ...
      'String','click here');
```

creates a pushbutton in the center of a figure window and returns a handle to the new object. But, so far, clicking on the button does nothing. The statement

```
s = 'set(b,''Position'',[.8*rand .9*rand .2 .1])';
```

creates a string containing a command that alters the pushbutton's position. Repeated execution of

```
eval(s)
```

moves the button to random positions. Finally,

```
set(b,'Callback',s)
```

installs `s` as the button's callback action, so every time you click on the button, it moves to a new position.

Graphical User Interface Design Tools

MATLAB includes a set of layout tools that simplify the process of creating graphical user interfaces (GUIs). These tools include:

- Layout Editor – add and arrange objects in the figure window.
- Alignment Tool – align objects with respect to each other.
- Property Inspector – inspect and set property values.
- Object Browser – observe a hierarchical list of the Handle Graphics objects in the current MATLAB session.
- Menu Editor – create window menus and context menus.

Access these tools from the Layout Editor. To start the Layout Editor, use the `guide` command. For example,

```
guide
```

displays an empty layout.

To load an existing GUI for editing, type (the `.fig` is not required)

```
guide mygui.fig
```

or use **Open...** from the **File** menu on the Layout Editor.

For More Information See “Creating Graphical User Interfaces” for more information.

Animations

MATLAB provides two ways of generating moving, animated graphics:

- Continually erase and then redraw the objects on the screen, making incremental changes with each redraw.
- Save a number of different pictures and then play them back as a movie.

Erase Mode Method

Using the `EraseMode` property is appropriate for long sequences of simple plots where the change from frame to frame is minimal. Here is an example showing simulated Brownian motion. Specify a number of points, such as

```
n = 20
```

and a temperature or velocity, such as

```
s = .02
```

The best values for these two parameters depend upon the speed of your particular computer. Generate `n` random points with (x,y) coordinates between $-1/2$ and $+1/2$.

```
x = rand(n,1)-0.5;
y = rand(n,1)-0.5;
```

Plot the points in a square with sides at -1 and +1. Save the handle for the vector of points and set its `EraseMode` to `xor`. This tells the MATLAB graphics system not to redraw the entire plot when the coordinates of one point are changed, but to restore the background color in the vicinity of the point using an “exclusive or” operation.

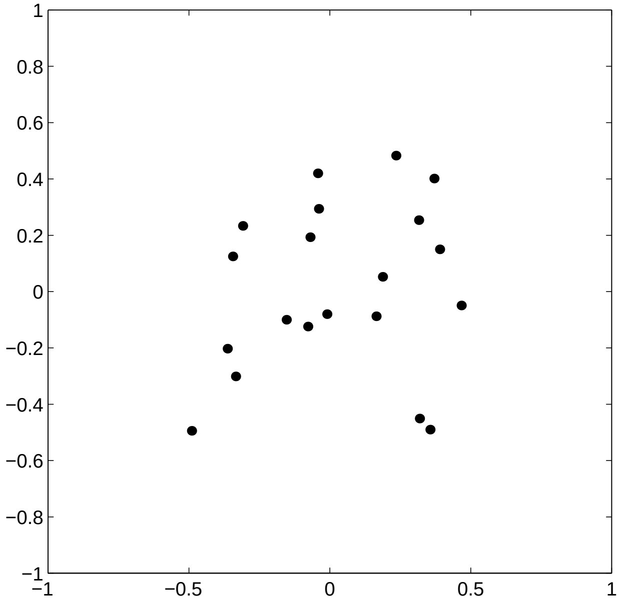
```
h = plot(x,y,'.');
axis([-1 1 -1 1])
axis square
grid off
set(h,'EraseMode','xor','MarkerSize',18)
```

Now begin the animation. Here is an infinite `while` loop, which you can eventually exit by typing **Ctrl+c**. Each time through the loop, add a small amount of normally distributed random noise to the coordinates of the points.

Then, instead of creating an entirely new plot, simply change the `XData` and `YData` properties of the original plot.

```
while 1
   drawnow
   x = x + s*randn(n,1);
   y = y + s*randn(n,1);
   set(h,'XData',x,'YData',y)
end
```

How long does it take for one of the points to get outside of the square? How long before all of the points are outside the square?



Creating Movies

If you increase the number of points in the Brownian motion example to something like `n = 300` and `s = .02`, the motion is no longer very fluid; it takes too much time to draw each time step. It becomes more effective to save a predetermined number of frames as bitmaps and to play them back as a *movie*.

First, decide on the number of frames, say

```
nframes = 50;
```

Next, set up the first plot as before, except using the default `EraseMode` (`normal`).

```
x = rand(n,1)-0.5;
y = rand(n,1)-0.5;
h = plot(x,y,'.');
set(h,'MarkerSize',18);
axis([-1 1 -1 1])
axis square
grid off
```

Generate the movie and use `getframe` to capture each frame.

```
for k = 1:nframes
   x = x + s*randn(n,1);
   y = y + s*randn(n,1);
   set(h,'XData',x,'YData',y)
   M(k) = getframe;
end
```

Finally, play the movie 30 times.

```
movie(M,30)
```

6

Programming with MATLAB

Flow Control

MATLAB has several flow control constructs:

- `if` statements
- `switch` statements
- `for` loops
- `while` loops
- `continue` statements
- `break` statements

For More Information See “Programming and Data Types” in Help for a complete discussion about programming in MATLAB.

---

if

The `if` statement evaluates a logical expression and executes a group of statements when the expression is *true*. The optional `elseif` and `else` keywords provide for the execution of alternate groups of statements. An `end` keyword, which matches the `if`, terminates the last group of statements. The groups of statements are delineated by the four keywords – no braces or brackets are involved.

MATLAB’s algorithm for generating a magic square of order *n* involves three different cases: when *n* is odd, when *n* is even but not divisible by 4, or when *n* is divisible by 4. This is described by

```
if rem(n,2) ~= 0
   M = odd_magic(n)
elseif rem(n,4) ~= 0
   M = single_even_magic(n)
else
   M = double_even_magic(n)
end
```

In this example, the three cases are mutually exclusive, but if they weren’t, the first *true* condition would be executed.

It is important to understand how relational operators and `if` statements work with matrices. When you want to check for equality between two variables, you might use

```
if A == B, ...
```

This is legal MATLAB code, and does what you expect when `A` and `B` are scalars. But when `A` and `B` are matrices, `A == B` does not test *if* they are equal, it tests *where* they are equal; the result is another matrix of 0's and 1's showing element-by-element equality. In fact, if `A` and `B` are not the same size, then `A == B` is an error.

The proper way to check for equality between two variables is to use the `isequal` function,

```
if isequal(A,B), ...
```

Here is another example to emphasize this point. If `A` and `B` are scalars, the following program will never reach the unexpected situation. But for most pairs of matrices, including our magic squares with interchanged columns, none of the matrix conditions `A > B`, `A < B` or `A == B` is true for *all* elements and so the `else` clause is executed.

```
if A > B
   'greater'
elseif A < B
   'less'
elseif A == B
   'equal'
else
   error('Unexpected situation')
end
```

Several functions are helpful for reducing the results of matrix comparisons to scalar conditions for use with `if`, including

```
isequal
isempty
all
any
```

switch and case

The `switch` statement executes groups of statements based on the value of a variable or expression. The keywords `case` and `otherwise` delineate the groups. Only the first matching case is executed. There must always be an `end` to match the `switch`.

The logic of the magic squares algorithm can also be described by

```
switch (rem(n,4)==0) + (rem(n,2)==0)
   case 0
      M = odd_magic(n)
   case 1
      M = single_even_magic(n)
   case 2
      M = double_even_magic(n)
   otherwise
      error('This is impossible')
end
```

Note Unlike the C language `switch` statement, MATLAB's `switch` does not fall through. If the first case statement is *true*, the other case statements do not execute. So, `break` statements are not required.

---

for

The `for` loop repeats a group of statements a fixed, predetermined number of times. A matching `end` delineates the statements.

```
for n = 3:32
   r(n) = rank(magic(n));
end
r
```

The semicolon terminating the inner statement suppresses repeated printing, and the `r` after the loop displays the final result.

It is a good idea to indent the loops for readability, especially when they are nested.

```
for i = 1:m
   for j = 1:n
      H(i,j) = 1/(i+j);
   end
end
```

while

The `while` loop repeats a group of statements an indefinite number of times under control of a logical condition. A matching `end` delineates the statements.

Here is a complete program, illustrating `while`, `if`, `else`, and `end`, that uses interval bisection to find a zero of a polynomial.

```
a = 0; fa = -Inf;
b = 3; fb = Inf;
while b-a > eps*b
   x = (a+b)/2;
   fx = x^3-2*x-5;
   if sign(fx) == sign(fa)
      a = x; fa = fx;
   else
      b = x; fb = fx;
   end
end
x
```

The result is a root of the polynomial $x^3 - 2x - 5$, namely

```
x =
   2.09455148154233
```

The cautions involving matrix comparisons that are discussed in the section on the `if` statement also apply to the `while` statement.

continue

The `continue` statement passes control to the next iteration of the `for` or `while` loop in which it appears, skipping any remaining statements in the body of the loop. In nested loops, continue passes control to the next iteration of the for or while loop enclosing it.

The example below shows a continue loop that counts the lines of code in the file, magic.m, skipping all blank lines and comments. A continue statement is used to advance to the next line in magic.m without incrementing the count whenever a blank line or comment line is encountered.

```
fid = fopen('magic.m','r');
count = 0;
while ~feof(fid)
    line = fgetl(fid);
    if isempty(line) | strncmp(line,'%',1)
        continue
    end
    count = count + 1;
end
disp(sprintf('%d lines',count));
```

break

The break statement lets you exit early from a for or while loop. In nested loops, break exits from the innermost loop only.

Here is an improvement on the example from the previous section. Why is this use of break a good idea?

```
a = 0; fa = -Inf;
b = 3; fb = Inf;
while b-a > eps*b
   x = (a+b)/2;
   fx = x^3-2*x-5;
   if fx == 0
      break
   elseif sign(fx) == sign(fa)
      a = x; fa = fx;
   else
      b = x; fb = fx;
   end
end
x
```

Other Data Structures

This section introduces you to some other data structures in MATLAB, including:

- Multidimensional arrays
- Cell arrays
- Characters and text
- Structures

For More Information For a complete discussion of MATLAB's data structures, see "Programming and Data Types" in Help.

Multidimensional Arrays

Multidimensional arrays in MATLAB are arrays with more than two subscripts. They can be created by calling `zeros`, `ones`, `rand`, or `randn` with more than two arguments. For example,

```
R = randn(3,4,5);
```

creates a 3-by-4-by-5 array with a total of 3x4x5 = 60 normally distributed random elements.

A three-dimensional array might represent three-dimensional physical data, say the temperature in a room, sampled on a rectangular grid. Or, it might represent a sequence of matrices, $A^{(k)}$, or samples of a time-dependent matrix, $A(t)$. In these latter cases, the (i, j)th element of the kth matrix, or the t_kth matrix, is denoted by `A(i,j,k)`.

MATLAB's and Dürer's versions of the magic square of order 4 differ by an interchange of two columns. Many different magic squares can be generated by interchanging columns. The statement

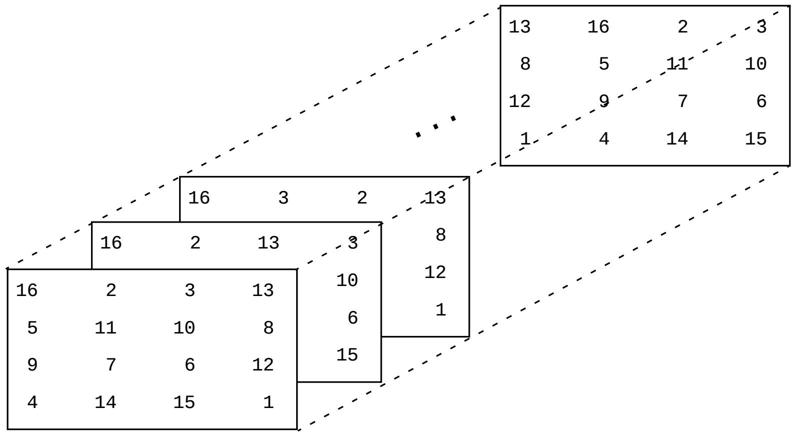
```
p = perms(1:4);
```

generates the 4! = 24 permutations of `1:4`. The `k`th permutation is the row vector, `p(k,:)`. Then

```
A = magic(4);
M = zeros(4,4,24);
for k = 1:24
   M(:,:,k) = A(:,p(k,:));
end
```

stores the sequence of 24 magic squares in a three-dimensional array, `M`. The size of `M` is

```
size(M)

ans =
     4     4    24
```



It turns out that the third matrix in the sequence is Dürer's.

```
M(:,:,3)

ans =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    15    14     1
```

The statement

```
sum(M,d)
```

computes sums by varying the dth subscript. So

```
sum(M,1)
```

is a 1-by-4-by-24 array containing 24 copies of the row vector

```
34    34    34    34
```

and

```
sum(M,2)
```

is a 4-by-1-by-24 array containing 24 copies of the column vector

```
34
34
34
34
```

Finally,

```
S = sum(M,3)
```

adds the 24 matrices in the sequence. The result has size 4-by-4-by-1, so it looks like a 4-by-4 array.

```
S =
   204   204   204   204
   204   204   204   204
   204   204   204   204
   204   204   204   204
```

Cell Arrays

Cell arrays in MATLAB are multidimensional arrays whose elements are copies of other arrays. A cell array of empty matrices can be created with the `cell` function. But, more often, cell arrays are created by enclosing a miscellaneous collection of things in curly braces, {}. The curly braces are also used with subscripts to access the contents of various cells. For example,

```
C = {A sum(A) prod(prod(A))}
```

produces a 1-by-3 cell array. The three cells contain the magic square, the row vector of column sums, and the product of all its elements. When `C` is displayed, you see

```
C =
     [4x4 double]    [1x4 double]    [20922789888000]
```

This is because the first two cells are too large to print in this limited space, but the third cell contains only a single number, 16!, so there is room to print it.

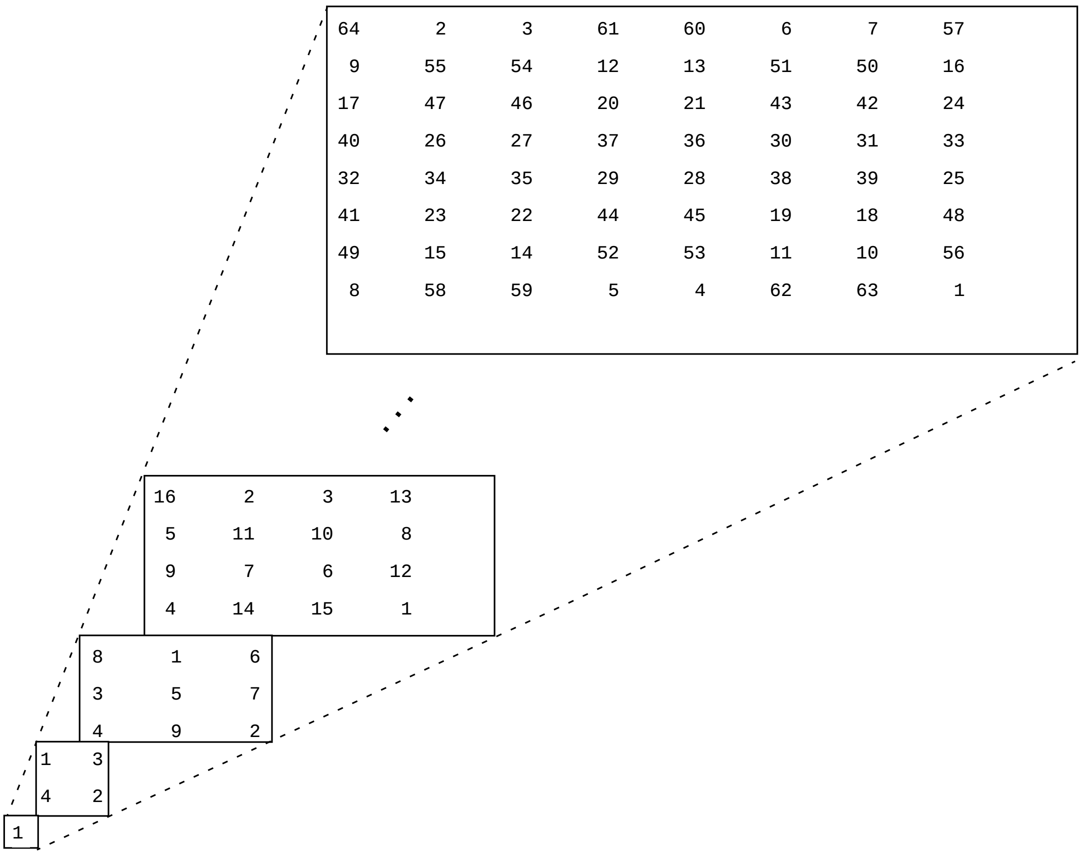
Here are two important points to remember. First, to retrieve the contents of one of the cells, use subscripts in curly braces. For example, `C{1}` retrieves the magic square and `C{3}` is 16!. Second, cell arrays contain *copies* of other arrays, not *pointers* to those arrays. If you subsequently change `A`, nothing happens to `C`.

Three-dimensional arrays can be used to store a sequence of matrices of the *same* size. Cell arrays can be used to store a sequence of matrices of *different* sizes. For example,

```
M = cell(8,1);
for n = 1:8
   M{n} = magic(n);
end
M
```

produces a sequence of magic squares of different order.

```
M =
     [          1]
     [ 2x2  double]
     [ 3x3  double]
     [ 4x4  double]
     [ 5x5  double]
     [ 6x6  double]
     [ 7x7  double]
     [ 8x8  double]
```



You can retrieve our old friend with

```
M{4}
```

Characters and Text

Enter text into MATLAB using single quotes. For example,

```
s = 'Hello'
```

The result is not the same kind of numeric matrix or array we have been dealing with up to now. It is a 1-by-5 character array.

Internally, the characters are stored as numbers, but not in floating-point format. The statement

```
a = double(s)
```

converts the character array to a numeric matrix containing floating-point representations of the ASCII codes for each character. The result is

```
a =
    72    101    108    108    111
```

The statement

```
s = char(a)
```

reverses the conversion.

Converting numbers to characters makes it possible to investigate the various fonts available on your computer. The printable characters in the basic ASCII character set are represented by the integers `32:127`. (The integers less than 32 represent nonprintable control characters.) These integers are arranged in an appropriate 6-by-16 array with

```
F = reshape(32:127,16,6)';
```

The printable characters in the extended ASCII character set are represented by `F+128`. When these integers are interpreted as characters, the result depends on the font currently being used. Type the statements

```
char(F)
char(F+128)
```

and then vary the font being used for the MATLAB Command Window. Select **Preferences** from the **File** menu. Be sure to try the **Symbol** and **Wingdings** fonts, if you have them on your computer. Here is one example of the kind of output you might obtain.

```
 !"#$%&'()*+,-./
0123456789:;<=>?
@ABCDEFGHIJKLMNO
PQRSTUVWXYZ[\]^_
`abcdefghijklmno
pqrstuvwxyz{|}~

† ¢£§•¶ß®©™´¨ ÆØ
    ¥        ªº æø
¿¡¬ ƒ   «»… ÀÃÕŒœ
-—“”‘’  ÿŸ/¤‹›ﬁﬂ
‡·‚„‰ÂÊÁËÈÍÎÏÌÓÔ
 ÒÚÛÙıˆ˜¯˘˙˚¸˝˛ˇ
```

Concatenation with square brackets joins text variables together into larger strings. The statement

```
h = [s, ' world']
```

joins the strings horizontally and produces

```
h =
   Hello world
```

The statement

```
v = [s; 'world']
```

joins the strings vertically and produces

```
v =
   Hello
   world
```

Note that a blank has to be inserted before the 'w' in `h` and that both words in `v` have to have the same length. The resulting arrays are both character arrays; `h` is 1-by-11 and `v` is 2-by-5.

To manipulate a body of text containing lines of different lengths, you have two choices – a padded character array or a cell array of strings. The `char` function accepts any number of lines, adds blanks to each line to make them all the same length, and forms a character array with each line in a separate row. For example,

```
S = char('A','rolling','stone','gathers','momentum.')
```

produces a 5-by-9 character array.

```
S =
A
rolling
stone
gathers
momentum.
```

There are enough blanks in each of the first four rows of S to make all the rows the same length. Alternatively, you can store the text in a cell array. For example,

```
C = {'A';'rolling';'stone';'gathers';'momentum.'}
```

is a 5-by-1 cell array.

```
C =
    'A'
    'rolling'
    'stone'
    'gathers'
    'momentum.'
```

You can convert a padded character array to a cell array of strings with

```
C = cellstr(S)
```

and reverse the process with

```
S = char(C)
```

Structures

Structures are multidimensional MATLAB arrays with elements accessed by textual *field designators*. For example,

```
S.name = 'Ed Plum';
S.score = 83;
S.grade = 'B+'
```

creates a scalar structure with three fields.

```
S =
     name: 'Ed Plum'
    score: 83
    grade: 'B+'
```

Like everything else in MATLAB, structures are arrays, so you can insert additional elements. In this case, each element of the array is a structure with several fields. The fields can be added one at a time,

```
S(2).name = 'Toni Miller';
S(2).score = 91;
S(2).grade = 'A-';
```

or, an entire element can be added with a single statement.

```
S(3) = struct('name','Jerry Garcia',...
               'score',70,'grade','C')
```

Now the structure is large enough that only a summary is printed.

```
S =
1x3 struct array with fields:
    name
    score
    grade
```

There are several ways to reassemble the various fields into other MATLAB arrays. They are all based on the notation of a *comma separated list*. If you type

```
S.score
```

it is the same as typing

```
S(1).score, S(2).score, S(3).score
```

This is a comma separated list. Without any other punctuation, it is not very useful. It assigns the three scores, one at a time, to the default variable ans and dutifully prints out the result of each assignment. But when you enclose the expression in square brackets,

```
[S.score]
```

it is the same as

```
[S(1).score, S(2).score, S(3).score]
```

which produces a numeric row vector containing all of the scores.

```
ans =
    83    91    70
```

Similarly, typing

```
S.name
```

just assigns the names, one at time, to ans. But enclosing the expression in curly braces,

```
{S.name}
```

creates a 1-by-3 cell array containing the three names.

```
ans =
    'Ed Plum'    'Toni Miller'    'Jerry Garcia'
```

And

```
char(S.name)
```

calls the char function with three arguments to create a character array from the name fields,

```
ans =
Ed Plum
Toni Miller
Jerry Garcia
```

Scripts and Functions

MATLAB is a powerful programming language as well as an interactive computational environment. Files that contain code in the MATLAB language are called M-files. You create M-files using a text editor, then use them as you would any other MATLAB function or command.

There are two kinds of M-files:

- Scripts, which do not accept input arguments or return output arguments. They operate on data in the workspace.
- Functions, which can accept input arguments and return output arguments. Internal variables are local to the function.

If you're a new MATLAB programmer, just create the M-files that you want to try out in the current directory. As you develop more of your own M-files, you will want to organize them into other directories and personal toolboxes that you can add to MATLAB's search path.

If you duplicate function names, MATLAB executes the one that occurs first in the search path.

To view the contents of an M-file, for example, `myfunction.m`, use

```
type myfunction
```

Scripts

When you invoke a *script*, MATLAB simply executes the commands found in the file. Scripts can operate on existing data in the workspace, or they can create new data on which to operate. Although scripts do not return output arguments, any variables that they create remain in the workspace, to be used in subsequent computations. In addition, scripts can produce graphical output using functions like `plot`.

For example, create a file called `magicrank.m` that contains these MATLAB commands.

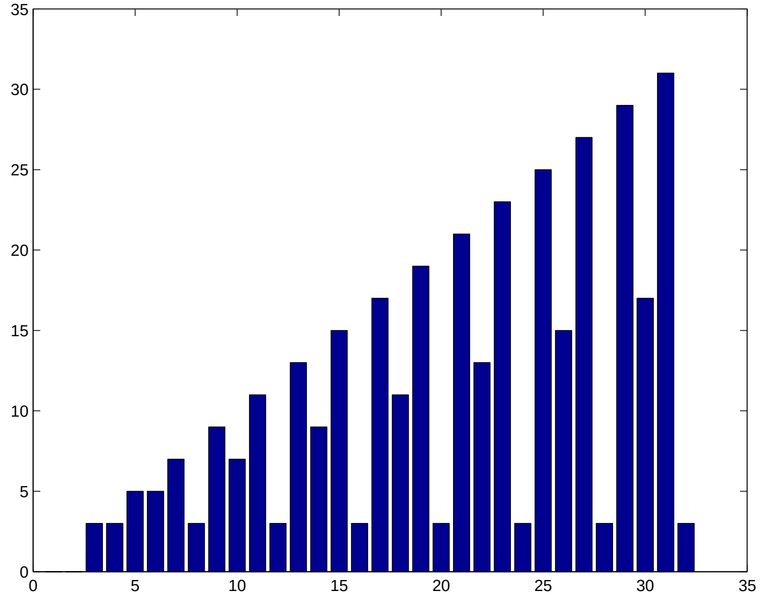
```
% Investigate the rank of magic squares
r = zeros(1,32);
for n = 3:32
   r(n) = rank(magic(n));
end
```

```
r
bar(r)
```

Typing the statement

```
magicrank
```

causes MATLAB to execute the commands, compute the rank of the first 30 magic squares, and plot a bar graph of the result. After execution of the file is complete, the variables `n` and `r` remain in the workspace.



Functions

Functions are M-files that can accept input arguments and return output arguments. The name of the M-file and of the function should be the same. Functions operate on variables within their own workspace, separate from the workspace you access at the MATLAB command prompt.

A good example is provided by `rank`. The M-file `rank.m` is available in the directory

```
toolbox/matlab/matfun
```

You can see the file with

```
type rank
```

Here is the file.

```
function r = rank(A,tol)
%   RANK Matrix rank.
%   RANK(A) provides an estimate of the number of linearly
%   independent rows or columns of a matrix A.
%   RANK(A,tol) is the number of singular values of A
%   that are larger than tol.
%   RANK(A) uses the default tol = max(size(A)) * norm(A) * eps.

s = svd(A);
if nargin==1
   tol = max(size(A)') * max(s) * eps;
end
r = sum(s > tol);
```

The first line of a function M-file starts with the keyword `function`. It gives the function name and order of arguments. In this case, there are up to two input arguments and one output argument.

The next several lines, up to the first blank or executable line, are comment lines that provide the help text. These lines are printed when you type

```
help rank
```

The first line of the help text is the H1 line, which MATLAB displays when you use the `lookfor` command or request `help` on a directory.

The rest of the file is the executable MATLAB code defining the function. The variable `s` introduced in the body of the function, as well as the variables on the first line, `r`, `A` and `tol`, are all *local* to the function; they are separate from any variables in the MATLAB workspace.

This example illustrates one aspect of MATLAB functions that is not ordinarily found in other programming languages – a variable number of arguments. The `rank` function can be used in several different ways.

```
rank(A)
r = rank(A)
r = rank(A,1.e-6)
```

Many M-files work this way. If no output argument is supplied, the result is stored in ans. If the second input argument is not supplied, the function computes a default value. Within the body of the function, two quantities named nargin and nargout are available which tell you the number of input and output arguments involved in each particular use of the function. The rank function uses nargin, but does not need to use nargout.

Global Variables

If you want more than one function to share a single copy of a variable, simply declare the variable as global in all the functions. Do the same thing at the command line if you want the base workspace to access the variable. The global declaration must occur before the variable is actually used in a function. Although it is not required, using capital letters for the names of global variables helps distinguish them from other variables. For example, create an M-file called falling.m.

```
function h = falling(t)
global GRAVITY
h = 1/2*GRAVITY*t.^2;
```

Then interactively enter the statements

```
global GRAVITY
GRAVITY = 32;
y = falling((0:.1:5)');
```

The two global statements make the value assigned to GRAVITY at the command prompt available inside the function. You can then modify GRAVITY interactively and obtain new solutions without editing any files.

Passing String Arguments to Functions

You can write MATLAB functions that accept string arguments without the parentheses and quotes. That is, MATLAB interprets

```
foo a b c
```

as

```
foo('a','b','c')
```

However, when using the unquoted form, MATLAB cannot return output arguments. For example,

```
legend apples oranges
```

creates a legend on a plot using the strings apples and oranges as labels. If you want the legend command to return its output arguments, then you must use the quoted form.

```
[legh,objh] = legend('apples','oranges');
```

In addition, you cannot use the unquoted form if any of the arguments are not strings.

Constructing String Arguments in Code

The quoted form enables you to construct string arguments within the code. The following example processes multiple data files, August1.dat, August2.dat, and so on. It uses the function int2str, which converts an integer to a character, to build the filename.

```
for d = 1:31
   s = ['August' int2str(d) '.dat'];
   load(s)
   % Code to process the contents of the d-th file
end
```

A Cautionary Note

While the unquoted syntax is convenient, it can be used incorrectly without causing MATLAB to generate an error. For example, given a matrix A,

```
A =
     0    -6    -1
     6     2   -16
    -5    20   -10
```

The eig command returns the eigenvalues of A.

```
eig(A)
ans =
  -3.0710
  -2.4645+17.6008i
  -2.4645-17.6008i
```

The following statement is not allowed because A is not a string, however MATLAB does not generate an error.

```
eig A
ans =
    65
```

MATLAB actually takes the eigenvalues of ASCII numeric equivalent of the letter A (which is the number 65).

The eval Function

The eval function works with text variables to implement a powerful text macro facility. The expression or statement

```
eval(s)
```

uses the MATLAB interpreter to evaluate the expression or execute the statement contained in the text string s.

The example of the previous section could also be done with the following code, although this would be somewhat less efficient because it involves the full interpreter, not just a function call.

```
for d = 1:31
   s = ['load August' int2str(d) '.dat'];
   eval(s)
   % Process the contents of the d-th file
end
```

Vectorization

To obtain the most speed out of MATLAB, it's important to vectorize the algorithms in your M-files. Where other programming languages might use `for` or `DO` loops, MATLAB can use vector or matrix operations. A simple example involves creating a table of logarithms.

```
x = .01;
for k = 1:1001
   y(k) = log10(x);
   x = x + .01;
end
```

A vectorized version of the same code is

```
x = .01:.01:10;
y = log10(x);
```

For more complicated code, vectorization options are not always so obvious. When speed is important, however, you should always look for ways to vectorize your algorithms.

Preallocation

If you can't vectorize a piece of code, you can make your `for` loops go faster by preallocating any vectors or arrays in which output results are stored. For example, this code uses the function `zeros` to preallocate the vector created in the `for` loop. This makes the `for` loop execute significantly faster.

```
r = zeros(32,1);
for n = 1:32
    r(n) = rank(magic(n));
end
```

Without the preallocation in the previous example, the MATLAB interpreter enlarges the `r` vector by one element each time through the loop. Vector preallocation eliminates this step and results in faster execution.

Function Handles

You can create a handle to any MATLAB function and then use that handle as a means of referencing the function. A function handle is typically passed in an argument list to other functions, which can then execute, or *evaluate*, the function using the handle.

Construct a function handle in MATLAB using the *at* sign, `@`, before the function name. The following example creates a function handle for the `sin` function and assigns it to the variable `fhandle`.

```
fhandle = @sin;
```

Evaluate a function handle using the MATLAB `feval` function. The function `plot_fhandle`, shown below, receives a function handle and data, and then performs an evaluation of the function handle on that data using `feval`.

```
function x = plot_fhandle(fhandle, data)
plot(data, feval(fhandle, data))
```

When you call `plot_fhandle` with a handle to the `sin` function and the argument shown below, the resulting evaluation produces a sine wave plot.

```
plot_fhandle(@sin, -pi:0.01:pi)
```

Function Functions

A class of functions, called “function functions,” works with nonlinear functions of a scalar variable. That is, one function works on another function. The function functions include:

- Zero finding
- Optimization
- Quadrature
- Ordinary differential equations

MATLAB represents the nonlinear function by a function M-file. For example, here is a simplified version of the function `humps` from the `matlab/demos` directory.

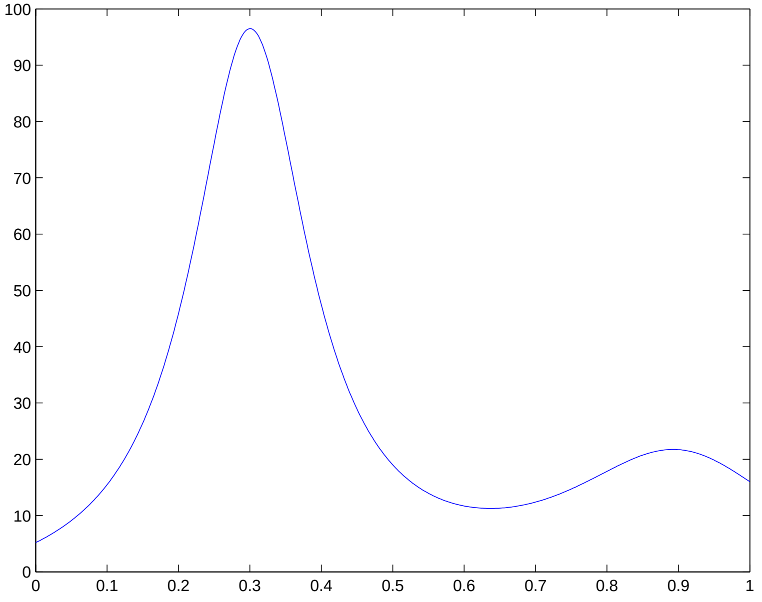
```
function y = humps(x)
y = 1./((x-.3).^2 + .01) + 1./((x-.9).^2 + .04) - 6;
```

Evaluate this function at a set of points in the interval $0 \le x \le 1$ with

```
x = 0:.002:1;
y = humps(x);
```

Then plot the function with

```
plot(x,y)
```



The graph shows that the function has a local minimum near x = 0.6. The function `fminsearch` finds the *minimizer*, the value of x where the function takes on this minimum. The first argument to `fminsearch` is a function handle to the function being minimized and the second argument is a rough guess at the location of the minimum.

```
p = fminsearch(@humps,.5)
p =
    0.6370
```

To evaluate the function at the minimizer,

```
humps(p)

ans =
   11.2528
```

Numerical analysts use the terms *quadrature* and *integration* to distinguish between numerical approximation of definite integrals and numerical integration of ordinary differential equations. MATLAB's quadrature routines are `quad` and `quadl`. The statement

```
Q = quadl(@humps,0,1)
```

computes the area under the curve in the graph and produces

```
Q =
   29.8583
```

Finally, the graph shows that the function is never zero on this interval. So, if you search for a zero with

```
z = fzero(@humps,.5)
```

you will find one outside of the interval

```
z =
   -0.1316
```

Demonstration Programs Included with MATLAB

MATLAB includes many demonstration programs that highlight various features and functions. For a complete list of the demos, at the command prompt type

```
help demos
```

To view a specific file, for example, `airfoil`, type

```
edit airfoil
```

To run a demonstration, type the filename at the command prompt. For example, to run the airfoil demonstration, type

```
airfoil
```

Note Many of the demonstrations use multiple windows and require you to press a key in the MATLAB Command Window to continue through the demonstration.

The following tables list some of the current demonstration programs that are available, organized into these categories:

- MATLAB Matrix Demonstration Programs
- MATLAB Numeric Demonstration Programs
- MATLAB Visualization Demonstration Programs
- MATLAB Language Demonstration Programs
- MATLAB Differential Equation Programs
- MATLAB Gallery Demonstration Programs
- MATLAB Game Demonstration Programs
- MATLAB Miscellaneous Demonstration Programs
- MATLAB Helper Functions Demonstration Programs

MATLAB Matrix Demonstration Programs

`airfoil`	Graphical demonstration of sparse matrix from NASA airfoil.
`buckydem`	Connectivity graph of the Buckminster Fuller geodesic dome.
`delsqdemo`	Finite difference Laplacian on various domains.
`eigmovie`	Symmetric eigenvalue movie.
`eigshow`	Graphical demonstration of matrix eigenvalues.
`intro`	Introduction to basic matrix operations in MATLAB.
`inverter`	Demonstration of the inversion of a large matrix.
`matmanip`	Introduction to matrix manipulation.
`rrefmovie`	Computation of reduced row echelon form.
`sepdemo`	Separators for a finite element mesh.
`sparsity`	Demonstration of the effect of sparsity orderings.
`svdshow`	Graphical demonstration of matrix singular values.

MATLAB Numeric Demonstration Programs

`bench`	MATLAB benchmark.
`census`	Prediction of the U.S. population in the year 2000.
`e2pi`	Two-dimensional, visual solution to the problem "Which is greater, e^{π} or π^{e}?"
`fftdemo`	Use of the FFT function for spectral analysis.
`fitdemo`	Nonlinear curve fit with simplex algorithm.
`fplotdemo`	Demonstration of plotting a function.

MATLAB Numeric Demonstration Programs (Continued)

`funfuns`	Demonstration of functions operating on other functions.
`lotkademo`	Example of ordinary differential equation solution.
`quaddemo`	Adaptive quadrature.
`qhulldemo`	Tessellation and interpolation of scattered data.
`quake`	Loma Prieta earthquake.
`spline2d`	Demonstration of `ginput` and `spline` in two dimensions.
`sunspots`	Demonstration of the fast Fourier transform (FFT) function in MATLAB used to analyze the variations in sunspot activity.
`zerodemo`	Zero finding with `fzero`.

MATLAB Visualization Demonstration Programs

`colormenu`	Demonstration of adding a colormap to the current figure.
`cplxdemo`	Maps of functions of a complex variable.
`earthmap`	Graphical demonstrations of earth's topography.
`graf2d`	Two-dimensional XY plots in MATLAB.
`graf2d2`	Three-dimensional XYZ plots in MATLAB.
`grafcplx`	Demonstration of complex function plots in MATLAB.
`imagedemo`	Demonstration of MATLAB's image capability.
`imageext`	Demonstration of changing and rotating image colormaps.

MATLAB Visualization Demonstration Programs (Continued)

`lorenz`	Graphical demonstration of the orbit around the Lorenz chaotic attractor.
`penny`	Several views of the penny data.
`vibes`	Vibrating L-shaped membrane movie.
`xfourier`	Graphical demonstration of Fourier series expansion.
`xpklein`	Klein bottle demo.
`xpsound`	Demonstration of MATLAB's sound capability.

MATLAB Language Demonstration Programs

`graf3d`	Demonstration of Handle Graphics for surface plots.
`hndlaxis`	Demonstration of Handle Graphics for axes.
`hndlgraf`	Demonstration of Handle Graphics for line plots.
`xplang`	Introduction to the MATLAB language.

MATLAB Differential Equation Programs

`amp1dae`	Stiff DAE from an electrical circuit.
`ballode`	Equations of motion for a bouncing ball used by `BALLDEMO`.
`brussode`	Stiff problem, modelling a chemical reaction (Brusselator).
`burgersode`	Burger's equation solved using a moving mesh technique.
`fem1ode`	Stiff problem with a time-dependent mass matrix.
`fem2ode`	Stiff problem with a time-independent mass matrix.

MATLAB Differential Equation Programs (Continued)

`hb1dae`	Stiff DAE from a conservation law.
`hb1ode`	Stiff problem 1 of Hindmarsh and Byrne.
`hb3ode`	Stiff problem 3 of Hindmarsh and Byrne.
`mat4bvp`	Find the fourth eigenvalue of the Mathieu's equation.
`odedemo`	Demonstration of the ODE suite integrators.
`odeexamples`	Browse the MATLAB ODE/DAE/BVP/PDE examples.
`orbitode`	Restricted 3 body problem used by `ORBITDEMO`.
`pdex1`	Example 1 for PDEPE.
`pdex2`	Example 2 for PDEPE.
`pdex3`	Example 3 for PDEPE.
`pdex4`	Example 4 for PDEPE.
`rigidode`	Euler equations of a rigid body without external forces.
`shockbvp`	The solution has a shock layer near x = 0.
`twobvp`	BVP that has exactly two solutions.
`vdpode`	Parameterizable van der Pol equation (stiff for large μ).

MATLAB Gallery Demonstration Programs

`cruller`	Graphical demonstration of a cruller.
`klein1`	Graphical demonstration of a Klein bottle.
`knot`	Tube surrounding a three-dimensional knot.
`logo`	Graphical demonstration of the MATLAB L-shaped membrane logo.

MATLAB Gallery Demonstration Programs (Continued)

modes	Graphical demonstration of 12 modes of the L-shaped membrane.
quivdemo	Graphical demonstration of the quiver function.
spharm2	Graphical demonstration of spherical surface harmonic.
tori4	Graphical demonstration of four-linked, unknotted tori.

MATLAB Game Demonstration Programs

fifteen	Sliding puzzle.
life	Conway's Game of Life.
soma	Soma cube.
xpbombs	Minesweeper game.

MATLAB Miscellaneous Demonstration Programs

chaingui	Matrix chain multiplication optimization.
codec	Alphabet transposition coder/decoder.
crulspin	Spinning cruller movie.
logospin	Movie of the MathWorks logo spinning.
makevase	Demonstration of a surface of revolution.
quatdemo	Quaternion rotation.
spinner	Colorful lines spinning through space.
travel	Traveling salesman problem.
truss	Animation of a bending bridge truss.

MATLAB Miscellaneous Demonstration Programs (Continued)

`wrldtrv`	Great circle flight routes around the globe.
`xphide`	Visual perception of objects in motion.
`xpquad`	Superquadrics plotting demonstration.

MATLAB Helper Functions Demonstration Programs

`bucky`	Graph of the Buckminster Fuller geodesic dome.
`cmdlnbgn`	Set up for command line demos.
`cmdlnend`	Clean up after command line demos.
`cmdlnwin`	Demo gateway routine for running command line demos.
`finddemo`	Command that finds available demos for individual toolboxes.
`helpfun`	Utility function for displaying help text conveniently.
`membrane`	The MathWorks logo.
`peaks`	Sample function of two variables.
`pltmat`	Command that displays a matrix in a figure window.

Getting More Information

The MathWorks Web site (`www.mathworks.com`) contains numerous M-files that have been written by users and MathWorks staff. These are accessible by selecting **Downloads**. Also, **Technical Notes**, which is accessible from our Technical Support Web site (`www.mathworks.com/support`), contains numerous examples on graphics, mathematics, API, Simulink, and others.

Symbolic Math Toolbox

Introduction

The Symbolic Math Toolbox incorporates symbolic computation into MATLAB's numeric environment. This toolbox supplements MATLAB's numeric and graphical facilities with several other types of mathematical computation.

Facility	Covers
Calculus	Differentiation, integration, limits, summation, and Taylor series
Linear Algebra	Inverses, determinants, eigenvalues, singular value decomposition, and canonical forms of symbolic matrices
Simplification	Methods of simplifying algebraic expressions
Solution of Equations	Symbolic and numerical solutions to algebraic and differential equations
Transforms	Fourier, Laplace, *z*-transform, and corresponding inverse transforms
Variable-Precision Arithmetic	Numerical evaluation of mathematical expressions to any specified accuracy

The computational engine underlying the toolboxes is the kernel of Maple, a system developed primarily at the University of Waterloo, Canada, and, more recently, at the Eidgenössiche Technische Hochschule, Zürich, Switzerland. Maple is marketed and supported by Waterloo Maple, Inc.

This version of the Symbolic Math Toolbox is designed to work with MATLAB 6 and Maple V Release 5.

The Symbolic Math Toolbox is a collection of more than one-hundred MATLAB functions that provide access to the Maple kernel using a syntax and style that is a natural extension of the MATLAB language. The toolbox also allows you to access functions in Maple's linear algebra package. With this toolbox, you can write your own M-files to access Maple functions and the Maple workspace.

The following sections of this tutorial provide explanation and examples on how to use the toolbox.

Section	Covers
"Getting Help"	How to get online help for Symbolic Math Toolbox functions
"Getting Started"	Basic symbolic math operations
"Calculus"	How to differentiate and integrate symbolic expressions
"Simplifications and Substitutions"	How to simplify and substitute values into expressions
"Variable-Precision Arithmetic"	How to control the precision of computations
"Linear Algebra"	Examples using the toolbox functions
"Solving Equations"	How to solve symbolic equations

For More Information You can access complete reference information for the Symbolic Math Toolbox functions from Help. Also, you can print the PDF version of the complete *Symbolic Math Toolbox User's Guide* (tutorial and reference information) from the Symbolic Math Toolbox roadmap in Help.

Getting Help

There are several ways to find information on using Symbolic Math Toolbox functions. One, of course, is to read this chapter! Another is to use online Help, which contains tutorials and reference information for all the functions. You can also use MATLAB's command line help system. Generally, you can obtain help on MATLAB functions simply by typing

```
help function
```

where *function* is the name of the MATLAB function for which you need help. This is not sufficient, however, for some Symbolic Math Toolbox functions. The reason? The Symbolic Math Toolbox "overloads" many of MATLAB's numeric functions. That is, it provides symbolic-specific implementations of the functions, using the same function name. To obtain help for the symbolic version of an overloaded function, type

```
help sym/function
```

where *function* is the overloaded function's name. For example, to obtain help on the symbolic version of the overloaded function, `diff`, type

```
help sym/diff
```

To obtain information on the numeric version, on the other hand, simply type

```
help diff
```

How can you tell whether a function is overloaded? The help for the numeric version tells you so. For example, the help for the `diff` function contains the section

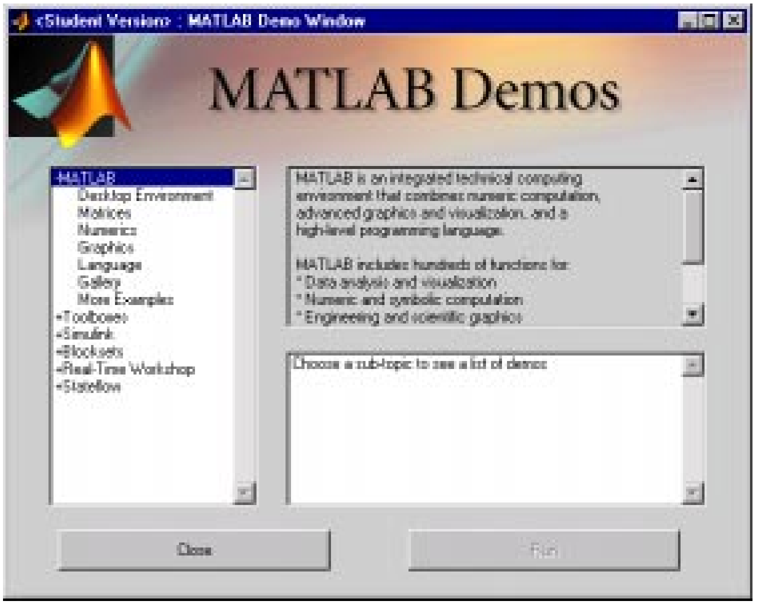
```
Overloaded methods
    help char/diff.m
    help sym/diff.m
```

This tells you that there are two other `diff` commands that operate on expressions of class `char` and class `sym`, respectively. See the next section for information on class `sym`. For more information on overloaded commands, see "Overloading Operators and Functions" in *Using MATLAB*, which is accessible from Help.

Getting Started

This section describes how to create and use symbolic objects. It also describes the default symbolic variable. If you are familiar with version 1 of the Symbolic Math Toolbox, please note that version 2 uses substantially different and simpler syntax.

To get a quick online introduction to the Symbolic Math Toolbox, type `demos` at the MATLAB command line. MATLAB displays the **MATLAB Demos** dialog box. Select **Symbolic Math** (in the left list box) and then **Introduction** (in the right list box).



Symbolic Objects

The Symbolic Math Toolbox defines a new MATLAB data type called a symbolic object or `sym` (for more information on data types, the MATLAB topic "Programming and Data Types" in *Using MATLAB*). Internally, a symbolic object is a data structure that stores a string representation of the symbol. The Symbolic Math Toolbox uses symbolic objects to represent symbolic variables, expressions, and matrices.

Creating Symbolic Variables and Expressions

The sym command lets you construct symbolic variables and expressions. For example, the commands

```
x = sym('x')
a = sym('alpha')
```

create a symbolic variable x that prints as x and a symbolic variable a that prints as alpha.

Suppose you want to use a symbolic variable to represent the golden ratio

$$\rho = \frac{1 + \sqrt{5}}{2}$$

The command

```
rho = sym('(1 + sqrt(5))/2')
```

achieves this goal. Now you can perform various mathematical operations on rho. For example,

```
f = rho^2 - rho - 1
```

returns

```
f =

(1/2+1/2*5^(1/2))^2-3/2-1/2*5^(1/2)
```

Then

```
simplify(f)
```

returns

```
0
```

Now suppose you want to study the quadratic function $f = ax^2 + bx + c$. The statement

```
f = sym('a*x^2 + b*x + c')
```

assigns the symbolic expression $ax^2 + bx + c$ to the variable f. Observe that in this case, the Symbolic Math Toolbox does not create variables corresponding to the terms of the expression, a, b, c, and x. To perform symbolic math operations (e.g., integration, differentiation, substitution, etc.) on f, you need to create the variables explicitly. You can do this by typing

```
a = sym('a')
b = sym('b')
c = sym('c')
x = sym('x')
```

or simply

```
syms a b c x
```

In general, you can use sym or syms to create symbolic variables. We recommend you use syms because it requires less typing.

Symbolic and Numeric Conversions

Consider the ordinary MATLAB quantity

```
t = 0.1
```

The sym function has four options for returning a symbolic representation of the numeric value stored in t. The 'f' option

```
sym(t,'f')
```

returns a symbolic floating-point representation

```
'1.999999999999a'*2^(-4)
```

The 'r' option

```
sym(t,'r')
```

returns the rational form

```
1/10
```

This is the default setting for sym. That is, calling sym without a second argument is the same as using sym with the 'r' option.

```
sym(t)

ans =
1/10
```

The third option 'e' returns the rational form of t plus the difference between the theoretical rational expression for t and its actual (machine) floating-point value in terms of eps (the floating-point relative accuracy).

```
sym(t,'e')

ans =
1/10+eps/40
```

The fourth option 'd' returns the decimal expansion of t up to the number of significant digits specified by digits.

```
sym(t,'d')

ans =
.10000000000000000555111512312578
```

The default value of digits is 32 (hence, sym(t,'d') returns a number with 32 significant digits), but if you prefer a shorter representation, use the digits command as follows.

```
digits(7)
sym(t,'d')

ans =
.1000000
```

A particularly effective use of sym is to convert a matrix from numeric to symbolic form. The command

```
A = hilb(3)
```

generates the 3-by-3 Hilbert matrix.

```
A =

    1.0000    0.5000    0.3333
    0.5000    0.3333    0.2500
    0.3333    0.2500    0.2000
```

By applying sym to A

```
A = sym(A)
```

you can obtain the (infinitely precise) symbolic form of the 3-by-3 Hilbert matrix.

```
A =

[   1, 1/2, 1/3]
[ 1/2, 1/3, 1/4]
[ 1/3, 1/4, 1/5]
```

Constructing Real and Complex Variables

The sym command allows you to specify the mathematical properties of symbolic variables by using the 'real' option. That is, the statements

```
x = sym('x','real'); y = sym('y','real');
```

or more efficiently

```
syms x y real
z = x + i*y
```

create symbolic variables x and y that have the added mathematical property of being real variables. Specifically this means that the expression

```
f = x^2 + y^2
```

is strictly nonnegative. Hence, z is a (formal) complex variable and can be manipulated as such. Thus, the commands

```
conj(x), conj(z), expand(z*conj(z))
```

return the complex conjugates of the variables

```
x, x-i*y, x^2+y^2
```

The conj command is the complex conjugate operator for the toolbox. If conj(x) == x returns 1, then x is a real variable.

To clear x of its “real” property, you must type

```
syms x unreal
```

or

```
x = sym('x','unreal')
```

The command

```
clear x
```

does *not* make x a nonreal variable.

Creating Abstract Functions

If you want to create an abstract (i.e., indeterminant) function $f(x)$, type

```
f = sym('f(x)')
```

Then f acts like $f(x)$ and can be manipulated by the toolbox commands. To construct the first difference ratio, for example, type

```
df = (subs(f,'x','x+h') - f)/'h'
```

or

```
syms x h
df = (subs(f,x,x+h)-f)/h
```

which returns

```
df =
(f(x+h)-f(x))/h
```

This application of sym is useful when computing Fourier, Laplace, and z-transforms.

Example: Creating a Symbolic Matrix

A circulant matrix has the property that each row is obtained from the previous one by cyclically permuting the entries one step forward. We create the circulant matrix A whose elements are a, b, and c, using the commands

```
syms a b c
A = [a b c; b c a; c a b]
```

which return

```
A =
[ a, b, c ]
[ b, c, a ]
[ c, a, b ]
```

Since A is circulant, the sum over each row and column is the same. Let's check this for the first row and second column. The command

```
sum(A(1,:))
```

returns

```
ans =
a+b+c
```

The command

```
sum(A(1,:)) == sum(A(:,2)) % This is a logical test.
```

returns

```
ans =
     1
```

Now replace the (2,3) entry of A with beta and the variable b with alpha. The commands

```
syms alpha beta;
A(2,3) = beta;
A = subs(A,b,alpha)
```

return

```
A =
[     a, alpha,     c]
[ alpha,     c,  beta]
[     c,     a, alpha]
```

From this example, you can see that using symbolic objects is very similar to using regular MATLAB numeric objects.

The Default Symbolic Variable

When manipulating mathematical functions, the choice of the independent variable is often clear from context. For example, consider the expressions in the table below.

Mathematical Function	MATLAB Command
$f = x^n$	`f = x^n`
$g = \sin(at + b)$	`g = sin(a*t + b)`
$h = J_v(z)$	`h = besselj(nu,z)`

If we ask for the derivatives of these expressions, without specifying the independent variable, then by mathematical convention we obtain $f' = nx^n$, $g' = a\cos(at + b)$, and $h' = J_v(z)(v/z) - J_{v+1}(z)$. Let's assume that the independent variables in these three expressions are x, t, and z, respectively. The other symbols, n, a, b, and v, are usually regarded as "constants" or "parameters." If, however, we wanted to differentiate the first expression with respect to n, for example, we could write

$$\frac{d}{dn}f(x) \text{ or } \frac{d}{dn}x^n$$

to get $x^n \ln x$.

By mathematical convention, independent variables are often lower-case letters found near the end of the Latin alphabet (e.g., *x*, *y*, or *z*). This is the idea behind `findsym`, a utility function in the toolbox used to determine default symbolic variables. Default symbolic variables are utilized by the calculus, simplification, equation-solving, and transform functions. To apply this utility to the example discussed above, type

```
syms a b n nu t x z
f = x^n; g = sin(a*t + b); h = besselj(nu,z);
```

This creates the symbolic expressions `f`, `g`, and `h` to match the example. To differentiate these expressions, we use `diff`.

```
diff(f)
```

returns

```
ans =
x^n*n/x
```

See the section “Differentiation” for a more detailed discussion of differentiation and the `diff` command.

Here, as above, we did not specify the variable with respect to differentiation. How did the toolbox determine that we wanted to differentiate with respect to `x`? The answer is the `findsym` command

```
findsym(f,1)
```

which returns

```
ans =
x
```

Similarly, `findsym(g,1)` and `findsym(h,1)` return `t` and `z`, respectively. Here the second argument of `findsym` denotes the number of symbolic variables we want to find in the symbolic object `f`, using the `findsym` rule (see below). The absence of a second argument in `findsym` results in a list of all symbolic variables in a given symbolic expression. We see this demonstrated below. The command

```
findsym(g)
```

returns the result

```
ans =
a, b, t
```

---

findsym Rule The default symbolic variable in a symbolic expression is the letter that is closest to `'x'` alphabetically. If there are two equally close, the letter later in the alphabet is chosen.

Here are some examples.

Expression	Variable Returned by findsym
x^n	x
sin(a*t+b)	t
besselj(nu,z)	z
w*y + v*z	y
exp(i*theta)	theta
log(alpha*x1)	x1
y*(4+3*i) + 6*j	y
sqrt(pi*alpha)	alpha

Creating Symbolic Math Functions

There are two ways to create functions:

- Use symbolic expressions
- Create an M-file

Using Symbolic Expressions

The sequence of commands

```
syms x y z
r = sqrt(x^2 + y^2 + z^2)
t = atan(y/x)
f = sin(x*y)/(x*y)
```

generates the symbolic expressions `r`, `t`, and `f`. You can use `diff`, `int`, `subs`, and other Symbolic Math Toolbox functions to manipulate such expressions.

Creating an M-File

M-files permit a more general use of functions. Suppose, for example, you want to create the sinc function sin(x)/x. To do this, create an M-file in the @sym directory.

```
function z = sinc(x)
%SINC The symbolic sinc function
%     sin(x)/x. This function
%     accepts a sym as the input argument.
if isequal(x,sym(0))
   z = 1;
else
   z = sin(x)/x;
end
```

You can extend such examples to functions of several variables. See the MATLAB topic “Programming and Data Types” in *Using MATLAB* for a more detailed discussion on object-oriented programming.

Calculus

The Symbolic Math Toolbox provides functions to do the basic operations of calculus; differentiation, limits, integration, summation, and Taylor series expansion. The following sections outline these functions.

Differentiation

Let's create a symbolic expression.

```
syms a x
f = sin(a*x)
```

Then

```
diff(f)
```

differentiates `f` with respect to its symbolic variable (in this case `x`), as determined by `findsym`.

```
ans =
cos(a*x)*a
```

To differentiate with respect to the variable `a`, type

```
diff(f,a)
```

which returns df/da

```
ans =
cos(a*x)*x
```

To calculate the second derivatives with respect to `x` and `a`, respectively, type

```
diff(f,2)
```

or

```
diff(f,x,2)
```

which return

```
ans =
-sin(a*x)*a^2
```

and

```
diff(f,a,2)
```

which returns

```
ans =
-sin(a*x)*x^2
```

Define a, b, x, n, t, and theta in the MATLAB workspace, using the sym command. The table below illustrates the diff command.

f	diff(f)
x^n	x^n*n/x
sin(a*t+b)	cos(a*t+b)*a
exp(i*theta)	i*exp(i*theta)

To differentiate the Bessel function of the first kind, besselj(nu,z), with respect to z, type

```
syms nu z
b = besselj(nu,z);
db = diff(b)
```

which returns

```
db =
-besselj(nu+1,z)+nu/z*besselj(nu,z)
```

The diff function can also take a symbolic matrix as its input. In this case, the differentiation is done element-by-element. Consider the example

```
syms a x
A = [cos(a*x),sin(a*x);-sin(a*x),cos(a*x)]
```

which returns

```
A =
[  cos(a*x),  sin(a*x)]
[ -sin(a*x),  cos(a*x)]
```

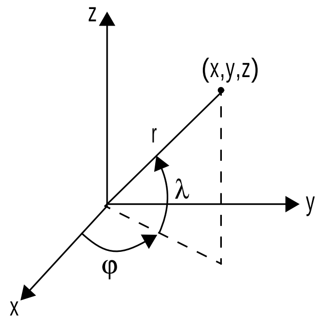
The command

```
diff(A)
```

returns

```
ans =
[ -sin(a*x)*a,  cos(a*x)*a]
[ -cos(a*x)*a, -sin(a*x)*a]
```

You can also perform differentiation of a column vector with respect to a row vector. Consider the transformation from Euclidean (x, y, z) to spherical (r, λ, φ) coordinates as given by $x = r\cos\lambda\cos\varphi$, $y = r\cos\lambda\sin\varphi$, and $z = r\sin\lambda$. Note that λ corresponds to elevation or latitude while φ denotes azimuth or longitude.



To calculate the Jacobian matrix, J, of this transformation, use the `jacobian` function. The mathematical notation for J is

$$J = \frac{\partial(x, y, x)}{\partial(r, \lambda, \varphi)}$$

For the purposes of toolbox syntax, we use `l` for λ and `f` for φ. The commands

```
syms r l f
x = r*cos(l)*cos(f); y = r*cos(l)*sin(f); z = r*sin(l);
J = jacobian([x; y; z], [r l f])
```

return the Jacobian

```
J =

[    cos(l)*cos(f), -r*sin(l)*cos(f), -r*cos(l)*sin(f)]
[    cos(l)*sin(f), -r*sin(l)*sin(f),  r*cos(l)*cos(f)]
[           sin(l),         r*cos(l),                0]
```

and the command

```
detJ = simple(det(J))
```

returns

```
detJ =
-cos(l)*r^2
```

Notice that the first argument of the `jacobian` function must be a column vector and the second argument a row vector. Moreover, since the determinant of the Jacobian is a rather complicated trigonometric expression, we used the `simple` command to make trigonometric substitutions and reductions (simplifications). The section "Simplifications and Substitutions" discusses simplification in more detail.

A table summarizing `diff` and `jacobian` follows.

Mathematical Operator	**MATLAB Command**
$\frac{df}{dx}$	`diff(f)` or `diff(f,x)`
$\frac{df}{da}$	`diff(f,a)`
$\frac{d^2 f}{db^2}$	`diff(f,b,2)`
$J = \frac{\partial(r, t)}{\partial(u, v)}$	`J = jacobian([r:t],[u,v])`

Limits

The fundamental idea in calculus is to make calculations on functions as a variable "gets close to" or approaches a certain value. Recall that the definition of the derivative is given by a limit

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided this limit exists. The Symbolic Math Toolbox allows you to compute the limits of functions in a direct manner. The commands

```
syms h n x
limit( (cos(x+h) - cos(x))/h,h,0 )
```

which return

```
ans =
-sin(x)
```

and

```
limit( (1 + x/n)^n,n,inf )
```

which returns

```
ans =
exp(x)
```

illustrate two of the most important limits in mathematics: the derivative (in this case of cos x) and the exponential function. While many limits

$$\lim_{x \to a} f(x)$$

are "two sided" (that is, the result is the same whether the approach is from the right or left of a), limits at the singularities of $f(x)$ are not. Hence, the three limits

$$\lim_{x \to 0} \frac{1}{x}, \quad \lim_{x \to 0-} \frac{1}{x}, \text{ and } \lim_{x \to 0+} \frac{1}{x}$$

yield the three distinct results: undefined, $-\infty$, and $+\infty$, respectively.

In the case of undefined limits, the Symbolic Math Toolbox returns `NaN` (not a number). The command

```
limit(1/x,x,0)
```

or

```
limit(1/x)
```

returns

```
ans =
NaN
```

The command

```
limit(1/x,x,0,'left')
```

returns

```
ans =
-inf
```

while the command

```
limit(1/x,x,0,'right')
```

returns

```
ans =
inf
```

Observe that the default case, `limit(f)` is the same as `limit(f,x,0)`. Explore the options for the `limit` command in this table. Here, we assume that `f` is a function of the symbolic object `x`.

Mathematical Operation	**MATLAB Command**
$\lim_{x \to 0} f(x)$	`limit(f)`
$\lim_{x \to a} f(x)$	`limit(f,x,a)` or `limit(f,a)`

Mathematical Operation	MATLAB Command
$\lim_{x \to a-} f(x)$	`limit(f,x,a,'left')`
$\lim_{x \to a+} f(x)$	`limit(f,x,a,'right')`

Integration

If `f` is a symbolic expression, then

```
int(f)
```

attempts to find another symbolic expression, `F`, so that `diff(F) = f`. That is, `int(f)` returns the indefinite integral or antiderivative of `f` (provided one exists in closed form). Similar to differentiation,

```
int(f,v)
```

uses the symbolic object `v` as the variable of integration, rather than the variable determined by `findsym`. See how `int` works by looking at this table.

Mathematical Operation	MATLAB Command
$\int x^n dx = \frac{x^{n+1}}{n+1}$	`int(x^n)` or `int(x^n,x)`
$\int_0^{\pi/2} \sin(2x)dx = 1$	`int(sin(2*x),0,pi/2)` or `int(sin(2*x),x,0,pi/2)`

$g = \cos(at+b)$
$\int g(t)dt = \sin(at+b)/a$
	`g = cos(a*t + b)`
	`int(g)` or
	`int(g,t)`

$\int J_1(z)dz = -J_0(z)$	`int(besselj(1,z))` or `int(besselj(1,z),z)`

In contrast to differentiation, symbolic integration is a more complicated task. A number of difficulties can arise in computing the integral. The antiderivative, F, may not exist in closed form; it may define an unfamiliar function; it may exist, but the software can't find the antiderivative; the software could find it on a larger computer, but runs out of time or memory on the available machine. Nevertheless, in many cases, MATLAB can perform symbolic integration successfully. For example, create the symbolic variables

```
syms a b theta x y n x1 u
```

This table illustrates integration of expressions containing those variables.

f	int(f)
x^n	x^(n+1)/(n+1)
y^(-1)	log(y)
n^x	1/log(n)*n^x
sin(a*theta+b)	-1/a*cos(a*theta+b)
exp(-x1^2)	1/2*pi^(1/2)*erf(x1)
1/(1+u^2)	atan(u)

The last example shows what happens if the toolbox can't find the antiderivative; it simply returns the command, including the variable of integration, unevaluated.

Definite integration is also possible. The commands

```
int(f,a,b)
```

and

```
int(f,v,a,b)
```

are used to find a symbolic expression for

$$\int_a^b f(x)dx \text{ and } \int_a^b f(v)dv$$

respectively.

Here are some additional examples.

f	a, b	int(f,a,b)
x^7	0, 1	1/8
1/x	1, 2	log(2)
log(x)*sqrt(x)	0, 1	-4/9
exp(-x^2)	0, inf	1/2*pi^(1/2)
besselj(1,z)	0, 1	1/4*hypergeom([1],[2, 2],-1/4)

For the Bessel function (`besselj`) example, it is possible to compute a numerical approximation to the value of the integral, using the `double` function. The command

```
a = int(besselj(1,z),0,1)
```

returns

```
a =
1/4*hypergeom([1],[2, 2],-1/4)
```

and the command

```
a = double(a)
```

returns

```
a =
0.2348
```

Integration with Real Constants

One of the subtleties involved in symbolic integration is the "value" of various parameters. For example, the expression

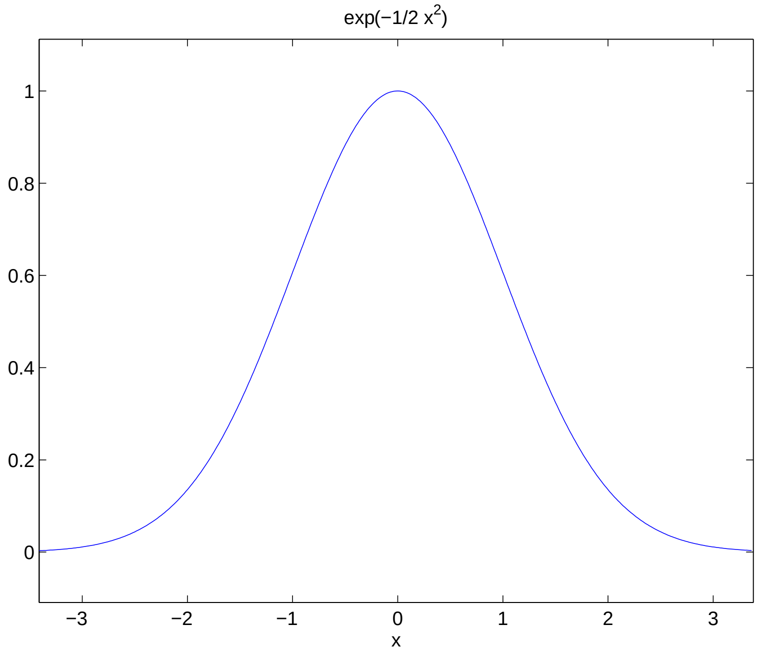
$$e^{-(kx)^2}$$

is the positive, bell shaped curve that tends to 0 as x tends to $\pm\infty$ for any real number k. An example of this curve is depicted below with

$$k = \frac{1}{\sqrt{2}}$$

and generated, using these commands.

```
syms x
k = sym(1/sqrt(2));
f = exp(-(k*x)^2);
ezplot(f)
```



The Maple kernel, however, does not, *a priori*, treat the expressions k^2 or x^2 as positive numbers. To the contrary, Maple assumes that the symbolic variables x and k as *a priori* indeterminate. That is, they are purely formal variables with no mathematical properties. Consequently, the initial attempt to compute the integral

$$\int_{-\infty}^{\infty} e^{-(kx)^2}\, dx$$

in the Symbolic Math Toolbox, using the commands

```
syms x k;
f = exp(-(k*x)^2);
int(f,x,-inf,inf)
```

results in the output

```
Definite integration: Can't determine if the integral is
convergent.
Need to know the sign of --> k^2
Will now try indefinite integration and then take limits.

Warning: Explicit integral could not be found.
ans =
int(exp(-k^2*x^2),x= -inf..inf)
```

In the next section, you will see how to make k a real variable and therefore k^2 positive.

Real Variables via sym

Notice that Maple is not able to determine the sign of the expression `k^2`. How does one surmount this obstacle? The answer is to make `k` a real variable, using the `sym` command. One particularly useful feature of `sym`, namely the `real` option, allows you to declare `k` to be a real variable. Consequently, the integral above is computed, in the toolbox, using the sequence

```
syms k real
int(f,x,-inf,inf)
```

which returns

```
ans =
signum(k)/k*pi^(1/2)
```

Notice that `k` is now a symbolic object in the MATLAB workspace and a real variable in the Maple kernel workspace. By typing

```
clear k
```

you only clear k in the MATLAB workspace. To ensure that k has no formal properties (that is, to ensure k is a purely formal variable), type

```
syms k unreal
```

This variation of the syms command clears k in the Maple workspace. You can also declare a sequence of symbolic variables w, y, x, z to be real, using

```
syms w x y z real
```

In this case, all of the variables in between the words syms and real are assigned the property real. That is, they are real variables in the Maple workspace.

Symbolic Summation

You can compute symbolic summations, when they exist, by using the symsum command. For example, the p-series

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$$

adds to $\pi^2/6$, while the geometric series $1 + x + x^2 + \ldots$ adds to $1/(1-x)$, provided $|x| < 1$. Three summations are demonstrated below.

```
syms x k
s1 = symsum(1/k^2,1,inf)
s2 = symsum(x^k,k,0,inf)

s1 =

1/6*pi^2

s2 =

-1/(x-1)
```

Taylor Series

The statements

```
syms x
f = 1/(5+4*cos(x))
T = taylor(f,8)
```

return

```
T =
1/9+2/81*x^2+5/1458*x^4+49/131220*x^6
```

which is all the terms up to, but not including, order eight $(O(x^8))$ in the Taylor series for $f(x)$.

$$\sum_{n=0}^{\infty} (x-a)^n \frac{f^{(n)}(a)}{n!}$$

Technically, `T` is a Maclaurin series, since its basepoint is `a = 0`.

The command

```
pretty(T)
```

prints `T` in a format resembling typeset mathematics.

```
              2            4     49    6
1/9 + 2/81 x  + 5/1458 x  + ------ x
                              131220
```

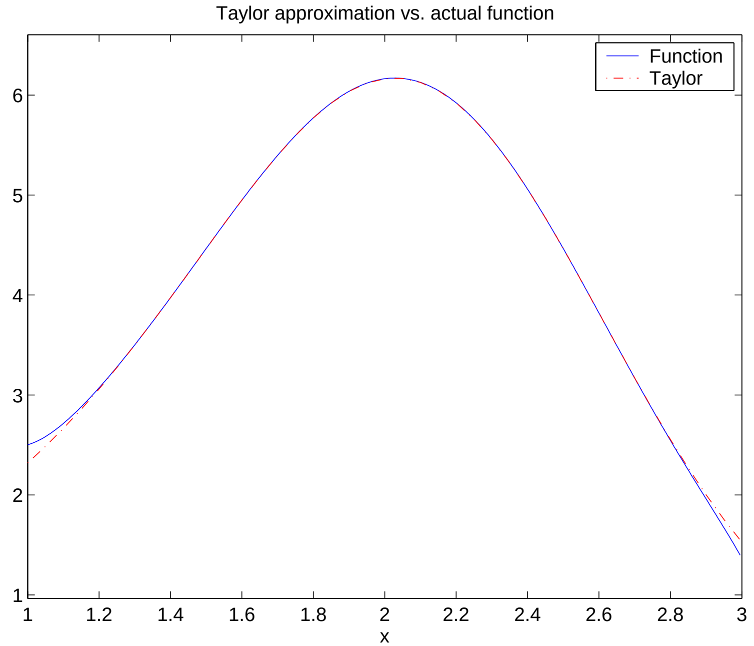
These commands

```
syms x
g = exp(x*sin(x))
t = taylor(g,12,2);
```

generate the first 12 nonzero terms of the Taylor series for `g` about `x = 2`.

Let's plot these functions together to see how well this Taylor approximation compares to the actual function `g`.

```
xd = 1:0.05:3; yd = subs(g,x,xd);
ezplot(t, [1,3]); hold on;
plot(xd, yd, 'r-.')
title('Taylor approximation vs. actual function');
legend('Function','Taylor')
```



Special thanks to Professor Gunnar Bäckstrøm of UMEA in Sweden for this example.

Extended Calculus Example

The function

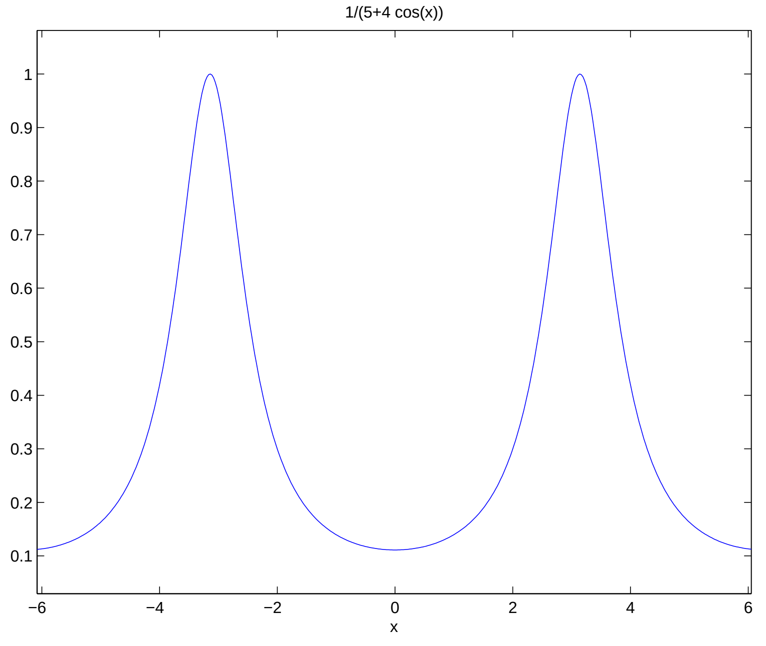
$$f(x) = \frac{1}{5 + 4\cos(x)}$$

provides a starting point for illustrating several calculus operations in the toolbox. It is also an interesting function in its own right. The statements

```
syms x
f = 1/(5+4*cos(x))
```

store the symbolic expression defining the function in `f`.

The function `ezplot(f)` produces the plot of $f(x)$ as shown below.



The `ezplot` function tries to make reasonable choices for the range of the x-axis and for the resulting scale of the y-axis. Its choices can be overridden by an additional input argument, or by subsequent `axis` commands. The default domain for a function displayed by `ezplot` is $-2\pi \leq x \leq 2\pi$. To produce a graph of $f(x)$ for $a \leq x \leq b$, type

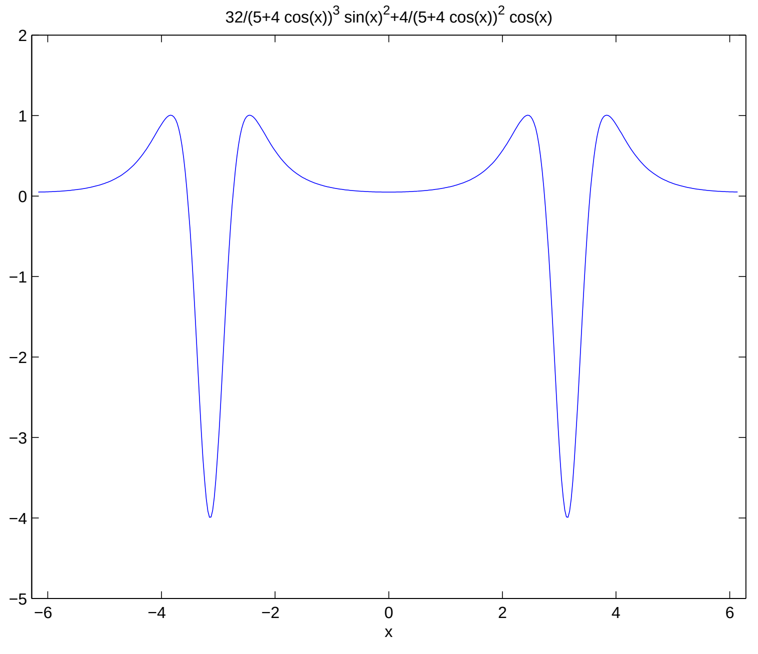
```
ezplot(f,[a b])
```

Let's now look at the second derivative of the function `f`.

```
f2 = diff(f,2)

f2 =
32/(5+4*cos(x))^3*sin(x)^2+4/(5+4*cos(x))^2*cos(x)
```

Equivalently, we can type `f2 = diff(f,x,2)`. The default scaling in `ezplot` cuts off part of `f2`'s graph. Set the axes limits manually to see the entire function.

```
ezplot(f2)
axis([-2*pi 2*pi -5 2])
```



From the graph, it appears that the values of $f''(x)$ lie between -4 and 1. As it turns out, this is not true. We can calculate the exact range for `f` (i.e., compute its actual maximum and minimum).

The actual maxima and minima of $f''(x)$ occur at the zeros of $f'''(x)$. The statements

```
f3 = diff(f2);
pretty(f3)
```

compute $f'''(x)$ and display it in a more readable format.

```
                   3
              sin(x)                sin(x) cos(x)             sin(x)
     384 --------------- + 96 --------------- - 4 ---------------
                        4                     3                   2
         (5 + 4 cos(x))       (5 + 4 cos(x))      (5 + 4 cos(x))
```

We can simplify this expression using the statements

```
f3 = simple(f3);
pretty(f3)
```

```
                               2                        2
        sin(x) (96 sin(x)  + 80 cos(x) + 80 cos(x)  - 25)
     4 -------------------------------------------------
                                        4
                          (5 + 4 cos(x))
```

Now use the `solve` function to find the zeros of $f'''(x)$.

```
z = solve(f3)
```

returns a 5-by-1 symbolic matrix

```
z =
[                                                    0]
[        atan((-255-60*19^(1/2))^(1/2),10+3*19^(1/2))]
[       atan(-(-255-60*19^(1/2))^(1/2),10+3*19^(1/2))]
[   atan((-255+60*19^(1/2))^(1/2)/(10-3*19^(1/2)))+pi]
[  -atan((-255+60*19^(1/2))^(1/2)/(10-3*19^(1/2)))-pi]
```

each of whose entries is a zero of $f'''(x)$. The commands

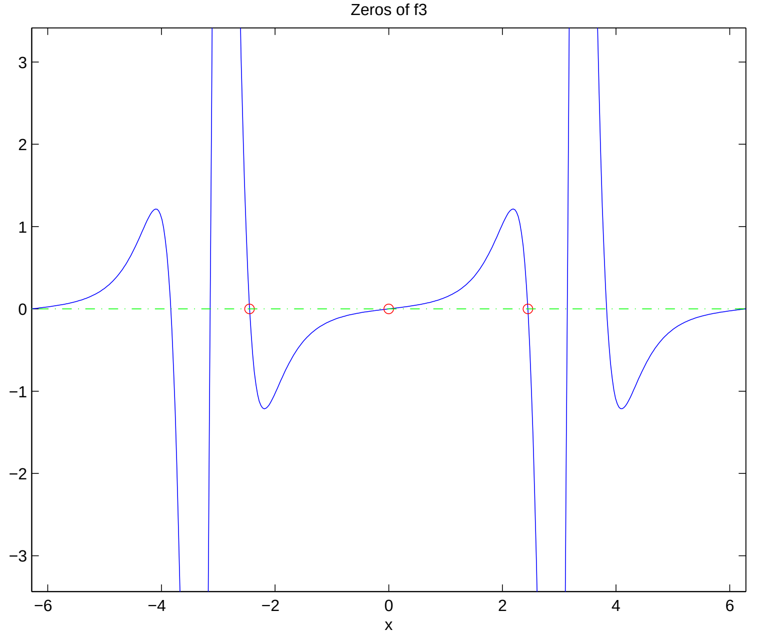
```
format; % Default format of 5 digits
zr = double(z)
```

convert the zeros to `double` form.

```
zr =

        0
        0+ 2.4381i
        0- 2.4381i
   2.4483
  -2.4483
```

So far, we have found three real zeros and two complex zeros. However, a graph of `f3` shows that we have not yet found all its zeros.

```
ezplot(f3)
hold on;
plot(zr,0*zr,'ro')
plot([-2*pi,2*pi], [0,0],'g-.');
title('Zeros of f3')
```



This occurs because $f'''(x)$ contains a factor of $\sin(x)$, which is zero at integer multiples of π. The function, `solve(sin(x))`, however, only reports the zero at $x = 0$.

We can obtain a complete list of the real zeros by translating `zr`

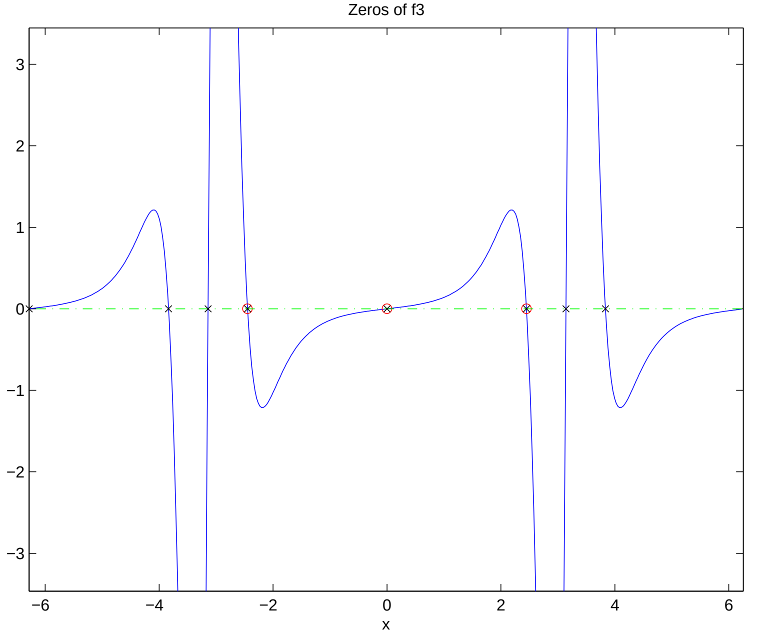
```
zr = [0 zr(4) pi 2*pi-zr(4)]
```

by multiples of 2π

```
zr = [zr-2*pi zr zr+2*pi];
```

Now let's plot the transformed `zr` on our graph for a complete picture of the zeros of `f3`.

```
plot(zr,0*zr,'kX')
```



The first zero of $f'''(x)$ found by `solve` is at $x = 0$. We substitute 0 for the symbolic variable in `f2`

```
f20 = subs(f2,x,0)
```

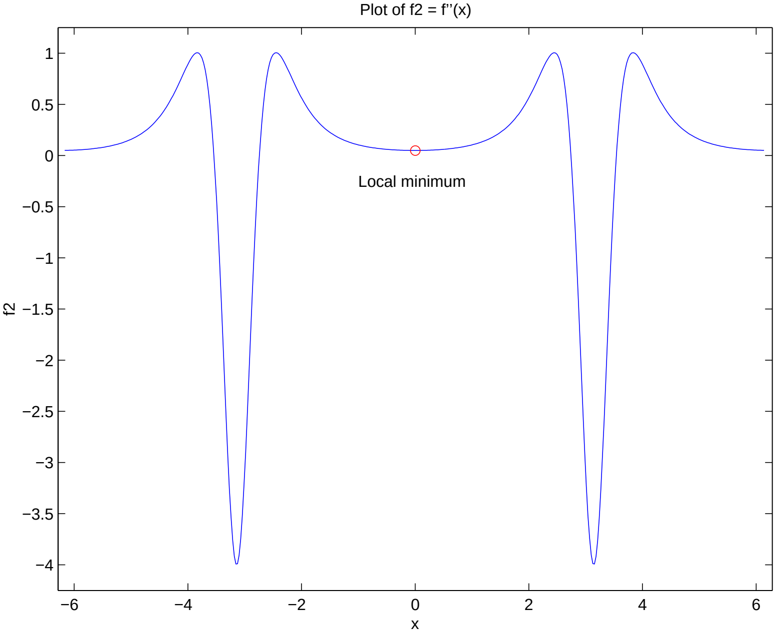
to compute the corresponding value of $f''(0)$.

```
f20 =
    0.0494
```

A look at the graph of $f''(x)$ shows that this is only a local minimum, which we demonstrate by replotting `f2`.

```
clf
ezplot(f2)
axis([-2*pi 2*pi -4.25 1.25])
ylabel('f2');
title('Plot of f2 = f''''(x)')
hold on
plot(0,double(f20),'ro')
text(-1,-0.25,'Local minimum')
```

The resulting plot



indicates that the global minima occur near $x = -\pi$ and $x = \pi$. We can demonstrate that they occur exactly at $x = \pm\pi$, using the following sequence of commands. First we try substituting $-\pi$ and π into $f'''(x)$.

```
simple([subs(f3,x,-sym(pi)),subs(f3,x,sym(pi))])
```

The result

```
ans =
[ 0, 0]
```

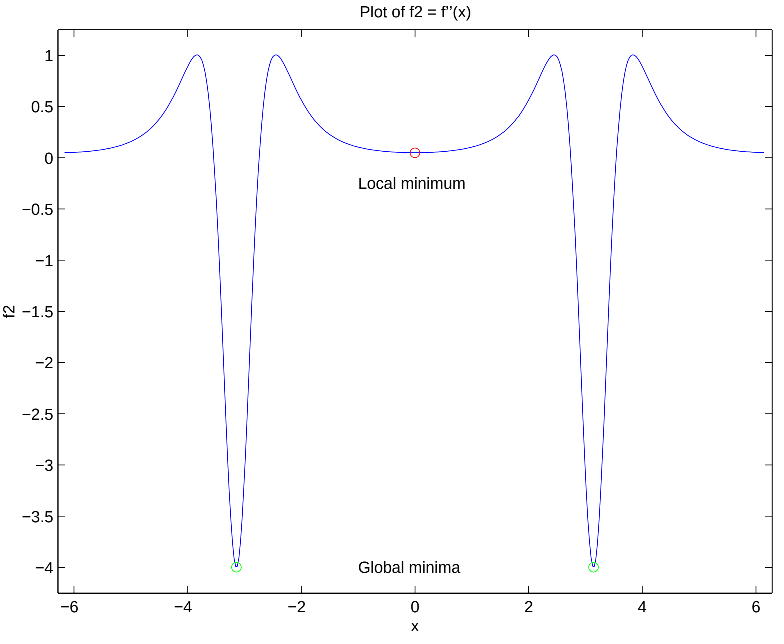
shows that $-\pi$ and π happen to be critical points of $f'''(x)$. We can see that $-\pi$ and π are global minima by plotting `f2(-pi)` and `f2(pi)` against `f2(x)`.

```
m1 = double(subs(f2,x,-pi)); m2 = double(subs(f2,x,pi));
plot(-pi,m1,'go',pi,m2,'go')
text(-1,-4,'Global minima')
```

The actual minima are m1, m2

```
ans =
[ -4, -4]
```

as shown in the following plot.



The foregoing analysis confirms part of our original guess that the range of $f''(x)$ is [-4, 1]. We can confirm the other part by examining the fourth zero of $f'''(x)$ found by solve. First extract the fourth zero from z and assign it to a separate variable

```
s = z(4)
```

to obtain

```
s =
atan((-255+60*19^(1/2))^(1/2)/(10-3*19^(1/2)))+pi
```

Executing

```
sd = double(s)
```

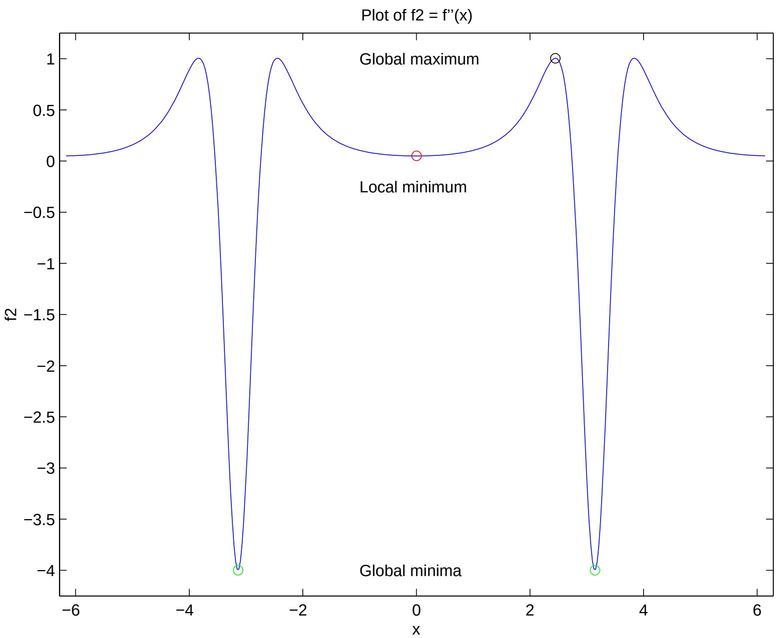
displays the zero's corresponding numeric value.

```
sd =
2.4483
```

Plotting the point `(s, f2(s))` against `f2`, using

```
M1 = double(subs(f2,x,s));
plot(sd,M1,'ko')
text(-1,1,'Global maximum')
```

visually confirms that `s` is a maximum.



The maximum is `M1 = 1.0051`.

Therefore, our guess that the maximum of $f''(x)$ is [-4, 1] was close, but incorrect. The actual range is [-4, 1.0051].

Now, let's see if integrating $f''(x)$ twice with respect to x recovers our original function $f(x) = 1/(5+4\cos x)$. The command

```
g = int(int(f2))
```

returns

```
g =
-8/(tan(1/2*x)^2+9)
```

This is certainly not the original expression for $f(x)$. Let's look at the difference $f(x) - g(x)$.

```
d = f - g
pretty(d)
```

```
                1                8
          ------------ + ---------------
          5 + 4 cos(x)             2
                         tan(1/2 x)  + 9
```

We can simplify this using `simple(d)` or `simplify(d)`. Either command produces

```
ans =
1
```

This illustrates the concept that differentiating $f(x)$ twice, then integrating the result twice, produces a function that may differ from $f(x)$ by a linear function of x.

Finally, integrate $f(x)$ once more.

```
F = int(f)
```

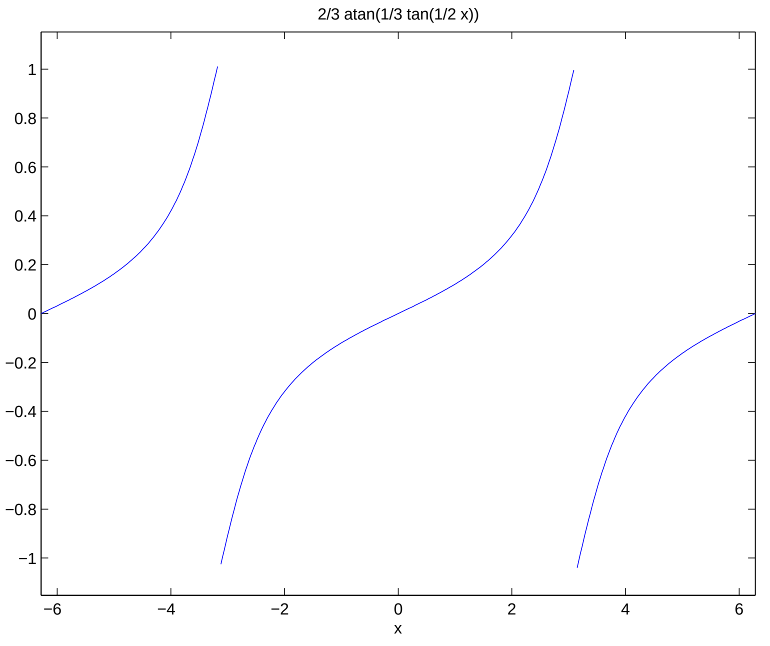
The result

```
F =
2/3*atan(1/3*tan(1/2*x))
```

involves the arctangent function.

Though $F(x)$ is the antiderivative of a continuous function, it is itself discontinuous as the following plot shows.

```
ezplot(F)
```

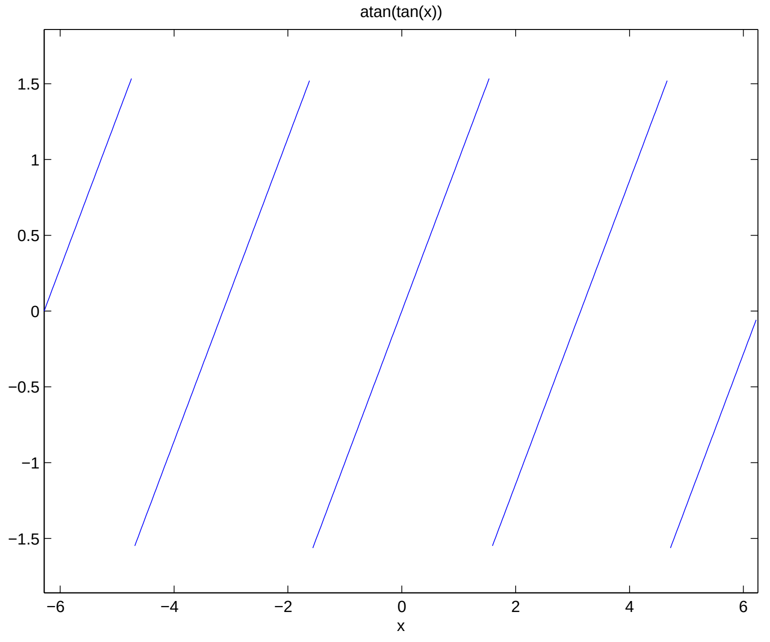


Note that $F(x)$ has jumps at $x = \pm\pi$. This occurs because $\tan x$ is singular at $x = \pm\pi$.

In fact, as

```
ezplot(atan(tan(x)))
```

shows, the numerical value of `atan(tan(x))` differs from `x` by a piecewise constant function that has jumps at odd multiples of $\pi/2$.



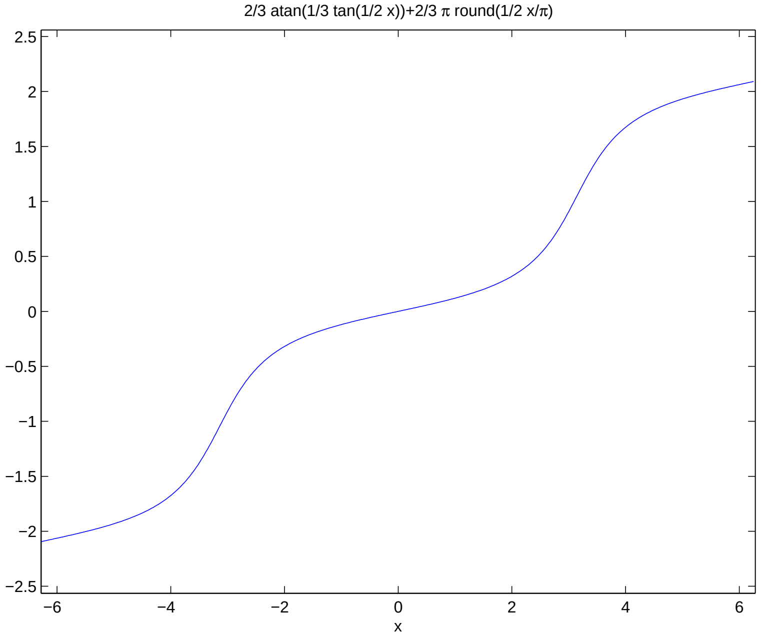
To obtain a representation of $F(x)$ that does not have jumps at these points, we must introduce a second function, $J(x)$, that compensates for the discontinuities. Then we add the appropriate multiple of $J(x)$ to $F(x)$

```
J = sym('round(x/(2*pi))');
c = sym('2/3*pi');
F1 = F+c*J
F1 =
2/3*atan(1/3*tan(1/2*x))+2/3*pi*round(1/2*x/pi)
```

and plot the result.

```
ezplot(F1,[-6.28,6.28])
```

This representation does have a continuous graph.



Notice that we use the domain [-6.28, 6.28] in `ezplot` rather than the default domain $[-2\pi, 2\pi]$. The reason for this is to prevent an evaluation of $F1 = 2/3\,\mathrm{atan}(1/3\tan 1/2x)$ at the singular points $x = -\pi$ and $x = \pi$ where the jumps in F and J do not cancel out one another. The proper handling of branch cut discontinuities in multivalued functions like arctan x is a deep and difficult problem in symbolic computation. Although MATLAB and Maple cannot do this entirely automatically, they do provide the tools for investigating such questions.

Simplifications and Substitutions

There are several functions that simplify symbolic expressions and are used to perform symbolic substitutions.

Simplifications

Here are three different symbolic expressions.

```
syms x
f = x^3-6*x^2+11*x-6
g = (x-1)*(x-2)*(x-3)
h = x*(x*(x-6)+11)-6
```

Here are their prettyprinted forms, generated by

```
pretty(f), pretty(g), pretty(h)
```

```
 3     2
x - 6 x  + 11 x - 6

(x - 1) (x - 2) (x - 3)

x (x (x - 6) + 11) - 6
```

These expressions are three different representations of the same mathematical function, a cubic polynomial in `x`.

Each of the three forms is preferable to the others in different situations. The first form, `f`, is the most commonly used representation of a polynomial. It is simply a linear combination of the powers of `x`. The second form, `g`, is the factored form. It displays the roots of the polynomial and is the most accurate for numerical evaluation near the roots. But, if a polynomial does not have such simple roots, its factored form may not be so convenient. The third form, `h`, is the Horner, or nested, representation. For numerical evaluation, it involves the fewest arithmetic operations and is the most accurate for some other ranges of `x`.

The symbolic simplification problem involves the verification that these three expressions represent the same function. It also involves a less clearly defined objective — which of these representations is “the simplest”?

This toolbox provides several functions that apply various algebraic and trigonometric identities to transform one representation of a function into another, possibly simpler, representation. These functions are `collect`, `expand`, `horner`, `factor`, `simplify`, and `simple`.

collect

The statement

```
collect(f)
```

views `f` as a polynomial in its symbolic variable, say `x`, and collects all the coefficients with the same power of `x`. A second argument can specify the variable in which to collect terms if there is more than one candidate. Here are a few examples.

f	collect(f)
`(x-1)*(x-2)*(x-3)`	`x^3-6*x^2+11*x-6`
`x*(x*(x-6)+11)-6`	`x^3-6*x^2+11*x-6`
`(1+x)*t + x*t`	`2*x*t+t`

expand

The statement

```
expand(f)
```

distributes products over sums and applies other identities involving functions of sums. For example,

f	expand(f)
`a*(x + y)`	`a*x + a*y`
`(x-1)*(x-2)*(x-3)`	`x^3-6*x^2+11*x-6`
`x*(x*(x-6)+11)-6`	`x^3-6*x^2+11*x-6`

f	expand(f)
`exp(a+b)`	`exp(a)*exp(b)`
`cos(x+y)`	`cos(x)*cos(y)-sin(x)*sin(y)`
`cos(3*acos(x))`	`4*x^3-3*x`

horner

The statement

```
horner(f)
```

transforms a symbolic polynomial `f` into its Horner, or nested, representation. For example,

f	horner(f)
`x^3-6*x^2+11*x-6`	`-6+(11+(-6+x)*x)*x`
`1.1+2.2*x+3.3*x^2`	`11/10+(11/5+33/10*x)*x`

factor

If `f` is a polynomial with rational coefficients, the statement

```
factor(f)
```

expresses `f` as a product of polynomials of lower degree with rational coefficients. If `f` cannot be factored over the rational numbers, the result is `f` itself. For example,

f	factor(f)
`x^3-6*x^2+11*x-6`	`(x-1)*(x-2)*(x-3)`
`x^3-6*x^2+11*x-5`	`x^3-6*x^2+11*x-5`
`x^6+1`	`(x^2+1)*(x^4-x^2+1)`

Here is another example involving `factor`. It factors polynomials of the form `x^n + 1`. This code

```
syms x;
n = (1:9)';
p = x.^n + 1;
f = factor(p);
[p, f]
```

returns a matrix with the polynomials in its first column and their factored forms in its second.

```
[                        x+1,                                 x+1 ]
[                      x^2+1,                               x^2+1 ]
[                      x^3+1,                     (x+1)*(x^2-x+1) ]
[                      x^4+1,                               x^4+1 ]
[                      x^5+1,             (x+1)*(x^4-x^3+x^2-x+1) ]
[                      x^6+1,                 (x^2+1)*(x^4-x^2+1) ]
[                      x^7+1, (x+1)*(1-x+x^2-x^3+x^4-x^5+x^6) ]
[                      x^8+1,                               x^8+1 ]
[                      x^9+1,       (x+1)*(x^2-x+1)*(x^6-x^3+1) ]
```

As an aside at this point, we mention that `factor` can also factor symbolic objects containing integers. This is an alternative to using the `factor` function in MATLAB's `specfun` directory. For example, the following code segment

```
N = sym(1);
for k = 2:11
   N(k) = 10*N(k-1)+1;
end
[N' factor(N')]
```

displays the factors of symbolic integers consisting of 1s.

```
[                      1,                      1]
[                     11,                   (11)]
[                    111,              (3)*(37)]
[                   1111,            (11)*(101)]
[                  11111,            (41)*(271)]
[                 111111, (3)*(7)*(11)*(13)*(37)]
[                1111111,          (239)*(4649)]
[               11111111,  (11)*(73)*(101)*(137)]
[              111111111,    (3)^2*(37)*(333667)]
[             1111111111, (11)*(41)*(271)*(9091)]
[            11111111111,       (513239)*(21649)]
```

simplify

The simplify function is a powerful, general purpose tool that applies a number of algebraic identities involving sums, integral powers, square roots and other fractional powers, as well as a number of functional identities involving trig functions, exponential and log functions, Bessel functions, hypergeometric functions, and the gamma function. Here are some examples.

f	simplify(f)
x*(x*(x-6)+11)-6	x^3-6*x^2+11*x-6
(1-x^2)/(1-x)	x+1
(1/a^3+6/a^2+12/a+8)^(1/3)	((2*a+1)^3/a^3)^(1/3)
syms x y positive log(x*y)	log(x)+log(y)
exp(x) * exp(y)	exp(x+y)
besselj(2,x) + besselj(0,x)	2/x*besselj(1,x)
gamma(x+1)-x*gamma(x)	0
cos(x)^2 + sin(x)^2	1

simple

The `simple` function has the unorthodox mathematical goal of finding a simplification of an expression that has the fewest number of characters. Of course, there is little mathematical justification for claiming that one expression is “simpler” than another just because its ASCII representation is shorter, but this often proves satisfactory in practice.

The `simple` function achieves its goal by independently applying `simplify`, `collect`, `factor`, and other simplification functions to an expression and keeping track of the lengths of the results. The `simple` function then returns the shortest result.

The `simple` function has several forms, each returning different output. The form

```
simple(f)
```

displays each trial simplification and the simplification function that produced it in the MATLAB command window. The `simple` function then returns the shortest result. For example, the command

```
simple(cos(x)^2 + sin(x)^2)
```

displays the following alternative simplifications in the MATLAB command window

```
simplify:
1

radsimp:
cos(x)^2+sin(x)^2

combine(trig):
1

factor:
cos(x)^2+sin(x)^2

expand:
cos(x)^2+sin(x)^2

convert(exp):
(1/2*exp(i*x)+1/2/exp(i*x))^2-1/4*(exp(i*x)-1/exp(i*x))^2
```

```
convert(sincos):
cos(x)^2+sin(x)^2


convert(tan):
(1-tan(1/2*x)^2)^2/(1+tan(1/2*x)^2)^2+4*tan(1/2*x)^2/
(1+tan(1/2*x)^2)^2

collect(x):
cos(x)^2+sin(x)^2
```

and returns

```
ans =
1
```

This form is useful when you want to check, for example, whether the shortest form is indeed the simplest. If you are not interested in how `simple` achieves its result, use the form

```
f = simple(f)
```

This form simply returns the shortest expression found. For example, the statement

```
f = simple(cos(x)^2+sin(x)^2)
```

returns

```
f =
1
```

If you want to know which simplification returned the shortest result, use the multiple output form.

```
[F, how] = simple(f)
```

This form returns the shortest result in the first variable and the simplification method used to achieve the result in the second variable. For example, the statement

```
[f, how] = simple(cos(x)^2+sin(x)^2)
```

returns

```
f =
1

how =
combine
```

The `simple` function sometimes improves on the result returned by `simplify`, one of the simplifications that it tries. For example, when applied to the examples given for `simplify`, `simple` returns a simpler (or at least shorter) result in two cases.

f	simplify(f)	simple(f)
`(1/a^3+6/a^2+12/a+8)^(1/3)`	`((2*a+1)^3/a^3)^(1/3)`	`(2*a+1)/a`
`syms x y positive` `log(x*y)`	`log(x)+log(y)`	`log(x*y)`

In some cases, it is advantageous to apply `simple` twice to obtain the effect of two different simplification functions. For example, the statements

```
f = (1/a^3+6/a^2+12/a+8)^(1/3);
simple(simple(f))
```

return

```
2+1/a
```

The first application, `simple(f)`, uses `radsimp` to produce `(2*a+1)/a`; the second application uses `combine(trig)` to transform this to `1/a+2`.

The `simple` function is particularly effective on expressions involving trigonometric functions. Here are some examples.

f	simple(f)
`cos(x)^2+sin(x)^2`	`1`
`2*cos(x)^2-sin(x)^2`	`3*cos(x)^2-1`
`cos(x)^2-sin(x)^2`	`cos(2*x)`

f	simple(f)
cos(x)+(-sin(x)^2)^(1/2)	cos(x)+i*sin(x)
cos(x)+i*sin(x)	exp(i*x)
cos(3*acos(x))	4*x^3-3*x

Substitutions

There are two functions for symbolic substitution: subexpr and subs.

subexpr

These commands

```
syms a x
s = solve(x^3+a*x+1)
```

solve the equation x^3+a*x+1 = 0 for x.

```
s =
[                     1/6*(-108+12*(12*a^3+81)^(1/2))^(1/3)-2*a/
                             (-108+12*(12*a^3+81)^(1/2))^(1/3)]
[ -1/12*(-108+12*(12*a^3+81)^(1/2))^(1/3)+a/
     (-108+12*(12*a^3+81)^(1/2))^(1/3)+1/2*i*3^(1/2)*(1/
     6*(-108+12*(12*a^3+81)^(1/2))^(1/3)+2*a/
     (-108+12*(12*a^3+81)^(1/2))^(1/3))]
[ -1/12*(-108+12*(12*a^3+81)^(1/2))^(1/3)+a/
     (-108+12*(12*a^3+81)^(1/2))^(1/3)-1/2*i*3^(1/2)*(1/
     6*(-108+12*(12*a^3+81)^(1/2))^(1/3)+2*a/
     (-108+12*(12*a^3+81)^(1/2))^(1/3))]
```

Use the pretty function to display s in a more readable form.

```
pretty(s)

s =
           [                       1/3       a                  ]
           [                 1/6 %1    - 2 -----                ]
           [                                  1/3               ]
           [                                %1                  ]
           [                                                    ]
           [         1/3     a           1/2 /      1/3       a  \]
           [- 1/12 %1    + ----- + 1/2 i 3    |1/6 %1    + 2 -----|]
           [                1/3               |                1/3|]
           [              %1                  \              %1   /]
           [                                                    ]
           [         1/3     a           1/2 /      1/3       a  \]
           [- 1/12 %1    + ----- - 1/2 i 3    |1/6 %1    + 2 -----|]
           [                1/3               |                1/3|]
           [              %1                  \              %1   /]

                                             3       1/2
                        %1 := -108 + 12 (12 a  + 81)
```

The `pretty` command inherits the `%n` (`n`, an integer) notation from Maple to denote subexpressions that occur multiple times in the symbolic object. The `subexpr` function allows you to save these common subexpressions as well as the symbolic object rewritten in terms of the subexpressions. The subexpressions are saved in a column vector called `sigma`.

Continuing with the example

```
r = subexpr(s)
```

returns

```
sigma =
-108+12*(12*a^3+81)^(1/2)

r =
[                              1/6*sigma^(1/3)-2*a/sigma^(1/3)]
[ -1/12*sigma^(1/3)+a/sigma^(1/3)+1/2*i*3^(1/2)*(1/6*sigma^
     (1/3)+2*a/sigma^(1/3))]
```

```
[ -1/12*sigma^(1/3)+a/sigma^(1/3)-1/2*i*3^(1/2)*(1/6*sigma^
     (1/3)+2*a/sigma^(1/3))]
```

Notice that subexpr creates the variable sigma in the MATLAB workspace. You can verify this by typing whos, or the command

```
 sigma
```

which returns

```
sigma =
-108+12*(12*a^3+81)^(1/2)
```

subs

Let's find the eigenvalues and eigenvectors of a circulant matrix A.

```
syms a b c
A = [a b c; b c a; c a b];
[v,E] = eig(A)

v =

[ -(a+(b^2-b*a-c*b-c*a+a^2+c^2)^(1/2)-b)/(a-c),
          -(a-(b^2-b*a-c*b-c*a+a^2+c^2)^(1/2)-b)/(a-c),  1]
[ -(b-c-(b^2-b*a-c*b-c*a+a^2+c^2)^(1/2))/(a-c),
          -(b-c+(b^2-b*a-c*b-c*a+a^2+c^2)^(1/2))/(a-c),  1]
[ 1,
         1,                                              1]


E =

[ (b^2-b*a-c*b-
   c*a+a^2+c^2)^(1/2),                     0,             0]
[                  0,   -(b^2-b*a-c*b-
                         c*a+a^2+c^2)^(1/2),             0]
[                  0,                      0,         b+c+a]
```

Suppose we want to replace the rather lengthy expression

```
(b^2-b*a-c*b-c*a+a^2+c^2)^(1/2)
```

throughout v and E. We first use subexpr

```
v = subexpr(v,'S')
```

which returns

```
S =
(b^2-b*a-c*b-c*a+a^2+c^2)^(1/2)

v =
[ -(a+S-b)/(a-c), -(a-S-b)/(a-c),               1]
[ -(b-c-S)/(a-c), -(b-c+S)/(a-c),               1]
[              1,              1,               1]
```

Next, substitute the symbol S into E with

```
E = subs(E,S,'S')

E =
[     S,      0,      0]
[     0,     -S,      0]
[     0,      0, b+c+a]
```

Now suppose we want to evaluate v at a = 10. We can do this using the subs command.

```
subs(v,a,10)
```

This replaces all occurrences of a in v with 10.

```
[ -(10+S-b)/(10-c), -(10-S-b)/(10-c),                 1]
[  -(b-c-S)/(10-c),  -(b-c+S)/(10-c),                 1]
[                1,                1,                 1]
```

Notice, however, that the symbolic expression represented by S is unaffected by this substitution. That is, the symbol a in S is not replaced by 10. The subs command is also a useful function for substituting in a variety of values for several variables in a particular expression. Let's look at S. Suppose that in addition to substituting a = 10, we also want to substitute the values for 2 and 10 for b and c, respectively. The way to do this is to set values for a, b, and c in the workspace. Then subs evaluates its input using the existing symbolic and double variables in the current workspace. In our example, we first set

```
a = 10; b = 2; c = 10;
subs(S)

ans =
8
```

To look at the contents of our workspace, type whos, which gives

```
Name      Size         Bytes  Class

  A         3x3            878  sym object
  E         3x3            888  sym object
  S         1x1            186  sym object
  a         1x1              8  double array
  ans       1x1            140  sym object
  b         1x1              8  double array
  c         1x1              8  double array
  v         3x3            982  sym object
```

a, b, and c are now variables of class double while A, E, S, and v remain symbolic expressions (class sym).

If you want to preserve a, b, and c as symbolic variables, but still alter their value within S, use this procedure.

```
syms a b c
subs(S,{a,b,c},{10,2,10})

ans =
8
```

Typing whos reveals that a, b, and c remain 1-by-1 sym objects.

The subs command can be combined with double to evaluate a symbolic expression numerically. Suppose we have

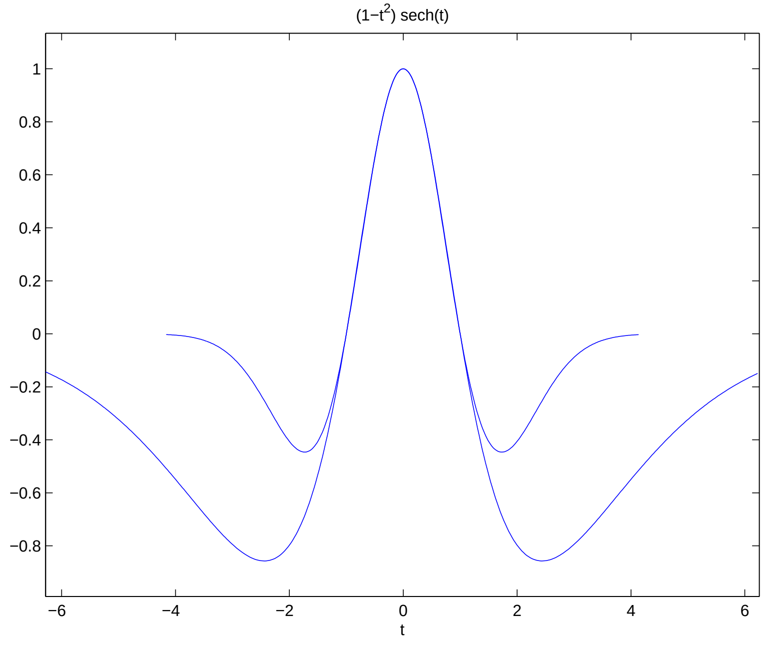
```
syms t
M = (1-t^2)*exp(-1/2*t^2);
P = (1-t^2)*sech(t);
```

and want to see how M and P differ graphically.

One approach is to type

```
ezplot(M); hold on; ezplot(P)
```

but this plot

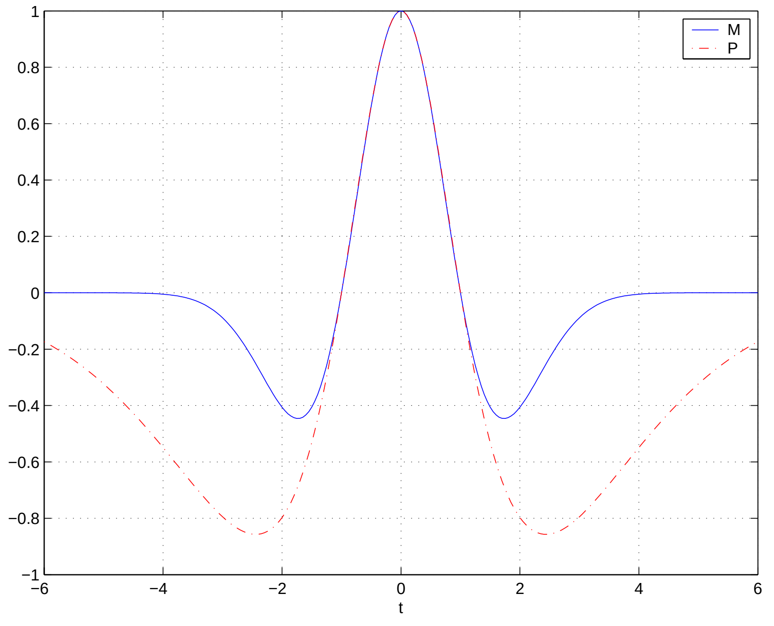


does not readily help us identify the curves.

Instead, combine subs, double, and plot

```
T = -6:0.05:6;
MT = double(subs(M,t,T));
PT = double(subs(P,t,T));
plot(T,MT,'b',T,PT,'r-.')
title(' ')
legend('M','P')
xlabel('t'); grid
```

to produce a multicolored graph that indicates the difference between M and P.



Finally the use of subs with strings greatly facilitates the solution of problems involving the Fourier, Laplace, or *z*-transforms.

Variable-Precision Arithmetic

Overview

There are three different kinds of arithmetic operations in this toolbox.

Numeric	MATLAB's floating-point arithmetic
Rational	Maple's exact symbolic arithmetic
VPA	Maple's variable-precision arithmetic

For example, the MATLAB statements

```
format long
1/2+1/3
```

use numeric computation to produce

```
0.83333333333333
```

With the Symbolic Math Toolbox, the statement

```
sym(1/2)+1/3
```

uses symbolic computation to yield

```
5/6
```

And, also with the toolbox, the statements

```
digits(25)
vpa('1/2+1/3')
```

use variable-precision arithmetic to return

```
.8333333333333333333333333
```

The floating-point operations used by numeric arithmetic are the fastest of the three, and require the least computer memory, but the results are not exact. The number of digits in the printed output of MATLAB's double quantities is controlled by the `format` statement, but the internal representation is always the eight-byte floating-point representation provided by the particular computer hardware.

In the computation of the numeric result above, there are actually three roundoff errors, one in the division of 1 by 3, one in the addition of 1/2 to the result of the division, and one in the binary to decimal conversion for the printed output. On computers that use IEEE floating-point standard arithmetic, the resulting internal value is the binary expansion of 5/6, truncated to 53 bits. This is approximately 16 decimal digits. But, in this particular case, the printed output shows only 15 digits.

The symbolic operations used by rational arithmetic are potentially the most expensive of the three, in terms of both computer time and memory. The results are exact, as long as enough time and memory are available to complete the computations.

Variable-precision arithmetic falls in between the other two in terms of both cost and accuracy. A global parameter, set by the function `digits`, controls the number of significant decimal digits. Increasing the number of digits increases the accuracy, but also increases both the time and memory requirements. The default value of `digits` is 32, corresponding roughly to floating-point accuracy.

The Maple documentation uses the term "hardware floating-point" for what we are calling "numeric" or "floating-point" and uses the term "floating-point arithmetic" for what we are calling "variable-precision arithmetic."

Example: Using the Different Kinds of Arithmetic

Rational Arithmetic

By default, the Symbolic Math Toolbox uses rational arithmetic operations, i.e., Maple's exact symbolic arithmetic. Rational arithmetic is invoked when you create symbolic variables using the `sym` function.

The `sym` function converts a double matrix to its symbolic form. For example, if the double matrix is

```
A =
1.1000    1.2000    1.3000
2.1000    2.2000    2.3000
3.1000    3.2000    3.3000
```

its symbolic form, `S = sym(A)`, is

```
S =
[11/10,  6/5, 13/10]
```

```
[21/10, 11/5, 23/10]
[31/10, 16/5, 33/10]
```

For this matrix A, it is possible to discover that the elements are the ratios of small integers, so the symbolic representation is formed from those integers. On the other hand, the statement

```
E = [exp(1) sqrt(2); log(3) rand]
```

returns a matrix

```
E =
2.71828182845905   1.41421356237310
1.09861228866811   0.21895918632809
```

whose elements are not the ratios of small integers, so sym(E) reproduces the floating-point representation in a symbolic form.

```
[3060513257434037*2^(-50), 3184525836262886*2^(-51)]
[2473854946935174*2^(-51), 3944418039826132*2^(-54)]
```

Variable-Precision Numbers

Variable-precision numbers are distinguished from the exact rational representation by the presence of a decimal point. A power of 10 scale factor, denoted by 'e', is allowed. To use variable-precision instead of rational arithmetic, create your variables using the vpa function.

For matrices with purely double entries, the vpa function generates the representation that is used with variable-precision arithmetic. Continuing on with our example, and using digits(4), applying vpa to the matrix S

```
vpa(S)
```

generates the output

```
S =
[1.100, 1.200, 1.300]
[2.100, 2.200, 2.300]
[3.100, 3.200, 3.300]
```

and with digits(25)

```
F = vpa(E)
```

generates

```
F =
[2.718281828459045534884808, 1.414213562373094923430017]
[1.098612288668110004152823, .2189591863280899719512718]
```

Converting to Floating-Point

To convert a rational or variable-precision number to its MATLAB floating-point representation, use the `double` function.

In our example, both `double(sym(E))` and `double(vpa(E))` return `E`.

Another Example

The next example is perhaps more interesting. Start with the symbolic expression

```
f = sym('exp(pi*sqrt(163))')
```

The statement

```
double(f)
```

produces the printed floating-point value

```
2.625374126407687e+17
```

Using the second argument of `vpa` to specify the number of digits,

```
vpa(f,18)
```

returns

```
262537412640768744.
```

whereas

```
vpa(f,25)
```

returns

```
262537412640768744.0000000
```

We suspect that `f` might actually have an integer value. This suspicion is reinforced by the 30 digit value, `vpa(f,30)`

```
262537412640768743.999999999999
```

Finally, the 40 digit value, `vpa(f,40)`

```
262537412640768743.9999999999992500725944
```

shows that `f` is very close to, but not exactly equal to, an integer.

Linear Algebra

Basic Algebraic Operations

Basic algebraic operations on symbolic objects are the same as operations on MATLAB objects of class `double`. This is illustrated in the following example.

The Givens transformation produces a plane rotation through the angle `t`. The statements

```
syms t;
G = [cos(t) sin(t); -sin(t) cos(t)]
```

create this transformation matrix.

```
G =
[  cos(t),  sin(t) ]
[ -sin(t),  cos(t) ]
```

Applying the Givens transformation twice should simply be a rotation through twice the angle. The corresponding matrix can be computed by multiplying `G` by itself or by raising `G` to the second power. Both

```
A = G*G
```

and

```
A = G^2
```

produce

```
A =
[cos(t)^2-sin(t)^2,   2*cos(t)*sin(t)]
[ -2*cos(t)*sin(t), cos(t)^2-sin(t)^2]
```

The `simple` function

```
A = simple(A)
```

uses a trigonometric identity to return the expected form by trying several different identities and picking the one that produces the shortest representation.

```
A =
[ cos(2*t), sin(2*t)]
[-sin(2*t), cos(2*t)]
```

A Givens rotation is an orthogonal matrix, so its transpose is its inverse. Confirming this by

```
I = G.' *G
```

which produces

```
I =
[cos(t)^2+sin(t)^2,                 0]
[                0, cos(t)^2+sin(t)^2]
```

and then

```
I = simple(I)
I =
[1, 0]
[0, 1]
```

Linear Algebraic Operations

Let's do several basic linear algebraic operations.

The command

```
H = hilb(3)
```

generates the 3-by-3 Hilbert matrix. With `format short`, MATLAB prints

```
H =
1.0000    0.5000    0.3333
0.5000    0.3333    0.2500
0.3333    0.2500    0.2000
```

The computed elements of `H` are floating-point numbers that are the ratios of small integers. Indeed, `H` is a MATLAB array of class `double`. Converting `H` to a symbolic matrix

```
H = sym(H)
```

gives

```
[  1, 1/2, 1/3]
[1/2, 1/3, 1/4]
[1/3, 1/4, 1/5]
```

This allows subsequent symbolic operations on `H` to produce results that correspond to the infinitely precise Hilbert matrix, `sym(hilb(3))`, not its floating-point approximation, `hilb(3)`. Therefore,

```
inv(H)
```

produces

```
[  9,  -36,   30]
[-36,  192, -180]
[ 30, -180,  180]
```

and

```
det(H)
```

yields

```
1/2160
```

We can use the backslash operator to solve a system of simultaneous linear equations. The commands

```
b = [1 1 1]'
x = H\b    % Solve Hx = b
```

produce the solution

```
[  3]
[-24]
[ 30]
```

All three of these results, the inverse, the determinant, and the solution to the linear system, are the exact results corresponding to the infinitely precise, rational, Hilbert matrix. On the other hand, using `digits(16)`, the command

```
V = vpa(hilb(3))
```

returns

```
[                1., .5000000000000000, .3333333333333333]
[.5000000000000000, .3333333333333333, .2500000000000000]
[.3333333333333333, .2500000000000000, .2000000000000000]
```

The decimal points in the representation of the individual elements are the signal to use variable-precision arithmetic. The result of each arithmetic operation is rounded to 16 significant decimal digits. When inverting the matrix, these errors are magnified by the matrix condition number, which for `hilb(3)` is about 500. Consequently,

```
inv(V)
```

which returns

```
[ 9.000000000000082, -36.00000000000039,  30.00000000000035]
[-36.00000000000039,  192.0000000000021, -180.0000000000019]
[ 30.00000000000035, -180.0000000000019,  180.0000000000019]
```

shows the loss of two digits. So does

```
det(V)
```

which gives

```
.462962962962958e-3
```

and

```
V\b
```

which is

```
[ 3.000000000000041]
[-24.00000000000021]
[ 30.00000000000019]
```

Since `H` is nonsingular, the null space of `H`

```
null(H)
```

and the column space of `H`

```
colspace(H)
```

produce an empty matrix and a permutation of the identity matrix, respectively. To make a more interesting example, let's try to find a value s for H(1,1) that makes H singular. The commands

```
syms s
H(1,1) = s
Z = det(H)
sol = solve(Z)
```

produce

```
H =
[  s, 1/2, 1/3]
[1/2, 1/3, 1/4]
[1/3, 1/4, 1/5]

Z =
1/240*s-1/270

sol =
8/9
```

Then

```
H = subs(H,s,sol)
```

substitutes the computed value of sol for s in H to give

```
H =
[8/9, 1/2, 1/3]
[1/2, 1/3, 1/4]
[1/3, 1/4, 1/5]
```

Now, the command

```
det(H)
```

returns

```
ans =
0
```

and

```
inv(H)
```

produces an error message

```
??? error using ==> inv
Error, (in inverse) singular matrix
```

because `H` is singular. For this matrix, `Z = null(H)` and `C = colspace(H)` are nontrivial.

```
Z =
[   1]
[  -4]
[10/3]

C =
[  0,     1]
[  1,     0]
[6/5, -3/10]
```

It should be pointed out that even though `H` is singular, `vpa(H)` is not. For any integer value `d`, setting

```
digits(d)
```

and then computing

```
det(vpa(H))
inv(vpa(H))
```

results in a determinant of size `10^(-d)` and an inverse with elements on the order of `10^d`.

Eigenvalues

The symbolic eigenvalues of a square matrix `A` or the symbolic eigenvalues and eigenvectors of `A` are computed, respectively, using the commands

```
E = eig(A)
[V,E] = eig(A)
```

The variable-precision counterparts are

```
E = eig(vpa(A))
[V,E] = eig(vpa(A))
```

The eigenvalues of A are the zeros of the characteristic polynomial of A, det(A-x*I), which is computed by

```
poly(A)
```

The matrix H from the last section provides our first example.

```
H =
[8/9, 1/2, 1/3]
[1/2, 1/3, 1/4]
[1/3, 1/4, 1/5]
```

The matrix is singular, so one of its eigenvalues must be zero. The statement

```
[T,E] = eig(H)
```

produces the matrices T and E. The columns of T are the eigenvectors of H.

```
T =

[    1, 28/153+2/153*12589^(1/2),  28/153-2/153*12589^(12)]
[   -4,                        1,                        1]
[ 10/3, 92/255-1/255*12589^(1/2), 292/255+1/255*12589^(12)]
```

Similarly, the diagonal elements of E are the eigenvalues of H.

```
E =

[0,                       0,                       0]
[0, 32/45+1/180*12589^(1/2),                       0]
[0,                       0, 32/45-1/180*12589^(1/2)]
```

It may be easier to understand the structure of the matrices of eigenvectors, T, and eigenvalues, E, if we convert T and E to decimal notation. We proceed as follows. The commands

```
Td = double(T)
Ed = double(E)
```

return

```
Td =
    1.0000    1.6497   -1.2837
   -4.0000    1.0000    1.0000
    3.3333    0.7051    1.5851
```

```
Ed =
     0         0         0
     0    1.3344         0
     0         0    0.0878
```

The first eigenvalue is zero. The corresponding eigenvector (the first column of `Td`) is the same as the basis for the null space found in the last section. The other two eigenvalues are the result of applying the quadratic formula to

```
x^2-64/45*x+253/2160
```

which is the quadratic factor in `factor(poly(H))`.

```
syms x
g = simple(factor(poly(H))/x);
solve(g)
```

Closed form symbolic expressions for the eigenvalues are possible only when the characteristic polynomial can be expressed as a product of rational polynomials of degree four or less. The Rosser matrix is a classic numerical analysis test matrix that happens to illustrate this requirement. The statement

```
R = sym(gallery('rosser'))
```

generates

```
 R =
[ 611   196  -192   407    -8   -52   -49    29]
[ 196   899   113  -192   -71   -43    -8   -44]
[-192   113   899   196    61    49     8    52]
[ 407  -192   196   611     8    44    59   -23]
[  -8   -71    61     8   411  -599   208   208]
[ -52   -43    49    44  -599   411   208   208]
[ -49    -8     8    59   208   208    99  -911]
[  29   -44    52   -23   208   208  -911    99]
```

The commands

```
p = poly(R);
pretty(factor(p))
```

produce

```
                2                   2                       2
 x (x - 1020) (x  - 1020 x + 100)(x  - 1040500) (x - 1000)
```

The characteristic polynomial (of degree 8) factors nicely into the product of two linear terms and three quadratic terms. We can see immediately that four of the eigenvalues are 0, 1020, and a double root at 1000. The other four roots are obtained from the remaining quadratics. Use

```
eig(R)
```

to find all these values

```
[                0]
[             1020]
[510+100*26^(1/2)]
[510-100*26^(1/2)]
[  10*10405^(1/2)]
[ -10*10405^(1/2)]
[             1000]
[             1000]
```

The Rosser matrix is not a typical example; it is rare for a full 8-by-8 matrix to have a characteristic polynomial that factors into such simple form. If we change the two “corner” elements of `R` from 29 to 30 with the commands

```
S = R;  S(1,8) = 30;  S(8,1) = 30;
```

and then try

```
p = poly(S)
```

we find

```
p =
40250968213600000+51264008540948000*x-
   1082699388411166000*x^2+4287832912719760*x^-3-
   5327831918568*x^4+82706090*x^5+5079941*x^6-
   4040*x^7+x^8
```

We also find that `factor(p)` is `p` itself. That is, the characteristic polynomial cannot be factored over the rationals.

For this modified Rosser matrix

```
F = eig(S)
```

returns

```
F =
[ -1020.053214255891516593189425260O]
[ -.17053529728768998575200874607757]
[   .21803980548301606860857564424981]
[  999.94691786044276755320289228602]
[  1000.1206982933841335712817075454]
[  1019.5243552632016358324933278291]
[  1019.9935501291629257348091808173]
[  1020.4201882015047278185457498840]
```

Notice that these values are close to the eigenvalues of the original Rosser matrix. Further, the numerical values of F are a result of Maple's floating-point arithmetic. Consequently, different settings of digits do not alter the number of digits to the right of the decimal place.

It is also possible to try to compute eigenvalues of symbolic matrices, but closed form solutions are rare. The Givens transformation is generated as the matrix exponential of the elementary matrix

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The Symbolic Math Toolbox commands

```
syms t
A = sym([0 1; -1 0]);
G = expm(t*A)
```

return

```
[  cos(t),  sin(t)]
[ -sin(t),  cos(t)]
```

Next, the command

```
g = eig(G)
```

produces

```
g =
[ cos(t)+(cos(t)^2-1)^(1/2)]
[ cos(t)-(cos(t)^2-1)^(1/2)]
```

We can use `simple` to simplify this form of `g`. Indeed, repeated application of `simple`

```
for  j = 1:4
   [g,how] = simple(g)
end
```

produces the best result

```
g =
[ cos(t)+(-sin(t)^2)^(1/2)]
[ cos(t)-(-sin(t)^2)^(1/2)]

how =
simplify

g =
[ cos(t)+i*sin(t)]
[ cos(t)-i*sin(t)]

how =
radsimp

g =
[   exp(i*t)]
[ 1/exp(i*t)]

how =
convert(exp)

g =
[  exp(i*t)]
[ exp(-i*t)]

how =
combine
```

Notice the first application of simple uses simplify to produce a sum of sines and cosines. Next, simple invokes radsimp to produce cos(t) + i*sin(t) for the first eigenvector. The third application of simple uses convert(exp) to change the sines and cosines to complex exponentials. The last application of simple uses simplify to obtain the final form.

Jordan Canonical Form

The Jordan canonical form results from attempts to diagonalize a matrix by a similarity transformation. For a given matrix A, find a nonsingular matrix V, so that inv(V)*A*V, or, more succinctly, J = V\A*V, is "as close to diagonal as possible." For almost all matrices, the Jordan canonical form is the diagonal matrix of eigenvalues and the columns of the transformation matrix are the eigenvectors. This always happens if the matrix is symmetric or if it has distinct eigenvalues. Some nonsymmetric matrices with multiple eigenvalues cannot be diagonalized. The Jordan form has the eigenvalues on its diagonal, but some of the superdiagonal elements are one, instead of zero. The statement

```
J = jordan(A)
```

computes the Jordan canonical form of A. The statement

```
[V,J] = jordan(A)
```

also computes the similarity transformation. The columns of V are the generalized eigenvectors of A.

The Jordan form is extremely sensitive to perturbations. Almost any change in A causes its Jordan form to be diagonal. This makes it very difficult to compute the Jordan form reliably with floating-point arithmetic. It also implies that A must be known exactly (i.e., without round-off error, etc.). Its elements must be integers, or ratios of small integers. In particular, the variable-precision calculation, jordan(vpa(A)), is not allowed.

For example, let

```
A = sym([12,32,66,116;-25,-76,-164,-294;
         21,66,143,256;-6,-19,-41,-73])
A =
[   12,   32,   66,  116]
[  -25,  -76, -164, -294]
[   21,   66,  143,  256]
[   -6,  -19,  -41,  -73]
```

Then

```
[V,J] = jordan(A)
```

produces

```
V =
[   4,  -2,   4,   3]
[  -6,   8, -11,  -8]
[   4,  -7,  10,   7]
[  -1,   2,  -3,  -2]

J =
[ 1, 1, 0, 0]
[ 0, 1, 0, 0]
[ 0, 0, 2, 1]
[ 0, 0, 0, 2]
```

Therefore A has a double eigenvalue at 1, with a single Jordan block, and a double eigenvalue at 2, also with a single Jordan block. The matrix has only two eigenvectors, V(:,1) and V(:,3). They satisfy

```
A*V(:,1) = 1*V(:,1)
A*V(:,3) = 2*V(:,3)
```

The other two columns of V are generalized eigenvectors of grade 2. They satisfy

```
A*V(:,2) = 1*V(:,2) + V(:,1)
A*V(:,4) = 2*V(:,4) + V(:,3)
```

In mathematical notation, with $\mathbf{v}_j$ = v(:,j), the columns of V and eigenvalues satisfy the relationships

$$(A - \lambda_2 I)\boldsymbol{v}_4 = \boldsymbol{v}_3$$

$$(A - \lambda_1 I)\boldsymbol{v}_2 = \boldsymbol{v}_1$$

Singular Value Decomposition

Only the variable-precision numeric computation of the singular value decomposition is available in the toolbox. One reason for this is that the formulas that result from symbolic computation are usually too long and complicated to be of much use. If A is a symbolic matrix of floating-point or variable-precision numbers, then

```
S = svd(A)
```

computes the singular values of A to an accuracy determined by the current setting of digits. And

```
[U,S,V] = svd(A);
```

produces two orthogonal matrices, U and V, and a diagonal matrix, S, so that

```
A = U*S*V';
```

Let's look at the n-by-n matrix A with elements defined by

```
A(i,j) = 1/(i-j+1/2)
```

For n = 5, the matrix is

```
[  2    -2   -2/3   -2/5   -2/7]
[2/3     2     -2   -2/3   -2/5]
[2/5   2/3      2     -2   -2/3]
[2/7   2/5    2/3      2     -2]
[2/9   2/7    2/5    2/3      2]
```

It turns out many of the singular values of these matrices are close to π.

The most obvious way of generating this matrix is

```
for i=1:n
    for j=1:n
      A(i,j) = sym(1/(i-j+1/2));
   end
end
```

The most efficient way to generate the matrix is

```
[J,I] = meshgrid(1:n);
A = sym(1./(I - J+1/2));
```

Since the elements of A are the ratios of small integers, vpa(A) produces a variable-precision representation, which is accurate to digits precision. Hence

```
S = svd(vpa(A))
```

computes the desired singular values to full accuracy. With n = 16 and digits(30), the result is

```
S =
[ 1.20968137605668985332455685357 ]
[ 2.69162158686066606774782763594 ]
[ 3.07790297231119748658424727354 ]
[ 3.13504054399744654843898901261 ]
[ 3.14106044663470063805218371924 ]
[ 3.14155754359918083691050658260 ]
[ 3.14159075458605848728982577119 ]
[ 3.14159256925492306470284863102 ]
[ 3.14159265052654880815569479613 ]
[ 3.14159265349961053143856838564 ]
[ 3.14159265358767361712392612384 ]
[ 3.14159265358975439206849907220 ]
[ 3.14159265358979270342635559051 ]
[ 3.14159265358979323325290142781 ]
[ 3.14159265358979323843066846712 ]
[ 3.14159265358979323846255035974 ]
```

There are two ways to compare S with pi, the floating-point representation of π. In the vector below, the first element is computed by subtraction with variable-precision arithmetic and then converted to a double. The second element is computed with floating-point arithmetic.

```
format short e
[double(pi*ones(16,1)-S)  pi-double(S)]
```

The results are

```
   1.9319e+00   1.9319e+00
   4.4997e-01   4.4997e-01
   6.3690e-02   6.3690e-02
   6.5521e-03   6.5521e-03
   5.3221e-04   5.3221e-04
   3.5110e-05   3.5110e-05
   1.8990e-06   1.8990e-06
```

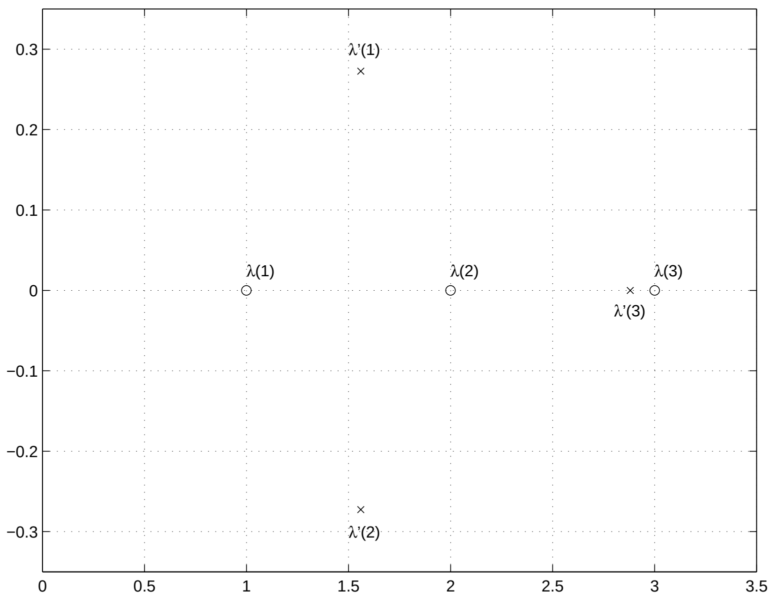
```
8.4335e-08   8.4335e-08
3.0632e-09   3.0632e-09
9.0183e-11   9.0183e-11
2.1196e-12   2.1196e-12
3.8846e-14   3.8636e-14
5.3504e-16   4.4409e-16
5.2097e-18            0
3.1975e-20            0
9.3024e-23            0
```

Since the relative accuracy of `pi` is `pi*eps`, which is `6.9757e-16`, either column confirms our suspicion that four of the singular values of the `16`-by-`16` example equal π to floating-point accuracy.

Eigenvalue Trajectories

This example applies several numeric, symbolic, and graphic techniques to study the behavior of matrix eigenvalues as a parameter in the matrix is varied. This particular setting involves numerical analysis and perturbation theory, but the techniques illustrated are more widely applicable.

In this example, we consider a 3-by-3 matrix A whose eigenvalues are 1, 2, 3. First, we perturb A by another matrix E and parameter t: $A \rightarrow A + tE$. As t increases from 0 to 10^{-6}, the eigenvalues $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 3$ change to $\lambda_1' \approx 1.5596 + 0.2726i$, $\lambda_2' \approx 1.5596 - 0.2726i$, $\lambda_3' \approx 2.8808$.



This, in turn, means that for some value of $t = \tau$, $0 < \tau < 10^{-6}$, the perturbed matrix $A(t) = A + tE$ has a double eigenvalue $\lambda_1 = \lambda_2$.

Let's find the value of `t`, called τ, where this happens.

The starting point is a MATLAB test example, known as `gallery(3)`.

```
A = gallery(3)
A =
  -149      -50     -154
   537      180      546
   -27       -9      -25
```

This is an example of a matrix whose eigenvalues are sensitive to the effects of roundoff errors introduced during their computation. The actual computed eigenvalues may vary from one machine to another, but on a typical workstation, the statements

```
format long
e = eig(A)
```

produce

```
e =
   0.99999999999642
   2.00000000000579
   2.99999999999780
```

Of course, the example was created so that its eigenvalues are actually 1, 2, and 3. Note that three or four digits have been lost to roundoff. This can be easily verified with the toolbox. The statements

```
B = sym(A);
e = eig(B)'
p = poly(B)
f = factor(p)
```

produce

```
e =
[1,  2,  3]

p =
x^3-6*x^2+11*x-6

f =
(x-1)*(x-2)*(x-3)
```

Are the eigenvalues sensitive to the perturbations caused by roundoff error because they are "close together"? Ordinarily, the values 1, 2, and 3 would be regarded as "well separated." But, in this case, the separation should be viewed on the scale of the original matrix. If `A` were replaced by `A/1000`, the eigenvalues, which would be .001, .002, .003, would "seem" to be closer together.

But eigenvalue sensitivity is more subtle than just "closeness." With a carefully chosen perturbation of the matrix, it is possible to make two of its eigenvalues coalesce into an actual double root that is extremely sensitive to roundoff and other errors.

One good perturbation direction can be obtained from the outer product of the left and right eigenvectors associated with the most sensitive eigenvalue. The following statement creates

```
E = [130,-390,0;43,-129,0;133,-399,0]
```

the perturbation matrix

```
E =
130  -390     0
 43  -129     0
133  -399     0
```

The perturbation can now be expressed in terms of a single, scalar parameter `t`. The statements

```
syms x t
A = A+t*E
```

replace `A` with the symbolic representation of its perturbation.

```
A =
[-149+130*t, -50-390*t, -154]
[  537+43*t, 180-129*t,  546]
[ -27+133*t,  -9-399*t,  -25]
```

Computing the characteristic polynomial of this new `A`

```
p = poly(A)
```

gives

```
p =
x^3-6*x^2+11*x-t*x^2+492512*t*x-6-1221271*t
```

Prettyprinting

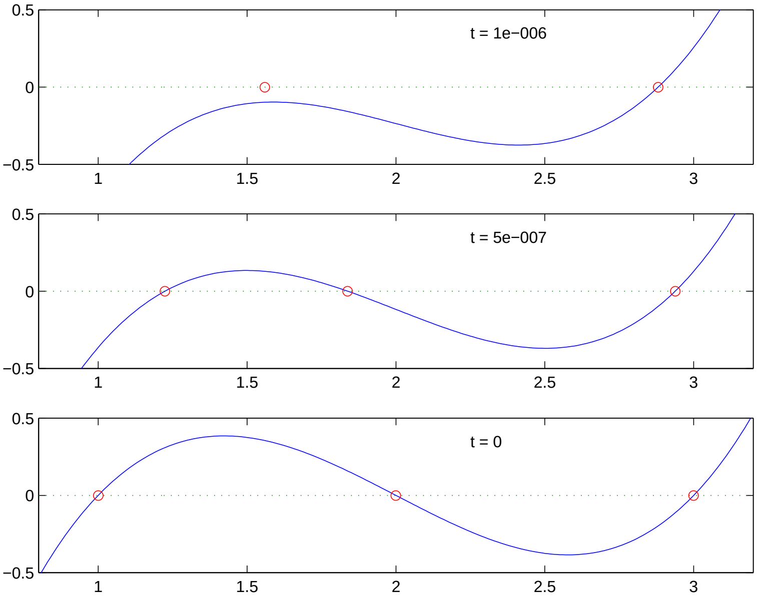
```
pretty(collect(p,x))
```

shows more clearly that `p` is a cubic in `x` whose coefficients vary linearly with `t`.

```
   3              2
  x  + (- t - 6) x  + (492512 t + 11) x - 6 - 1221271 t
```

It turns out that when `t` is varied over a very small interval, from 0 to 1.0e-6, the desired double root appears. This can best be seen graphically. The first figure shows plots of p, considered as a function of x, for three different values of t: t = 0, t = 0.5e-6, and t = 1.0e-6. For each value, the eigenvalues are computed numerically and also plotted.

```
x = .8:.01:3.2;
for k = 0:2
  c = sym2poly(subs(p,t,k*0.5e-6));
  y = polyval(c,x);
  lambda = eig(double(subs(A,t,k*0.5e-6)));
  subplot(3,1,3-k)
  plot(x,y,'-',x,0*x,':',lambda,0*lambda,'o')
  axis([.8 3.2 -.5 .5])
  text(2.25,.35,['t = ' num2str( k*0.5e-6 )]);
end
```



The bottom subplot shows the unperturbed polynomial, with its three roots at 1, 2, and 3. The middle subplot shows the first two roots approaching each other. In the top subplot, these two roots have become complex and only one real root remains.

The next statements compute and display the actual eigenvalues

```
e = eig(A);
pretty(e)
```

showing that `e(2)` and `e(3)` form a complex conjugate pair.

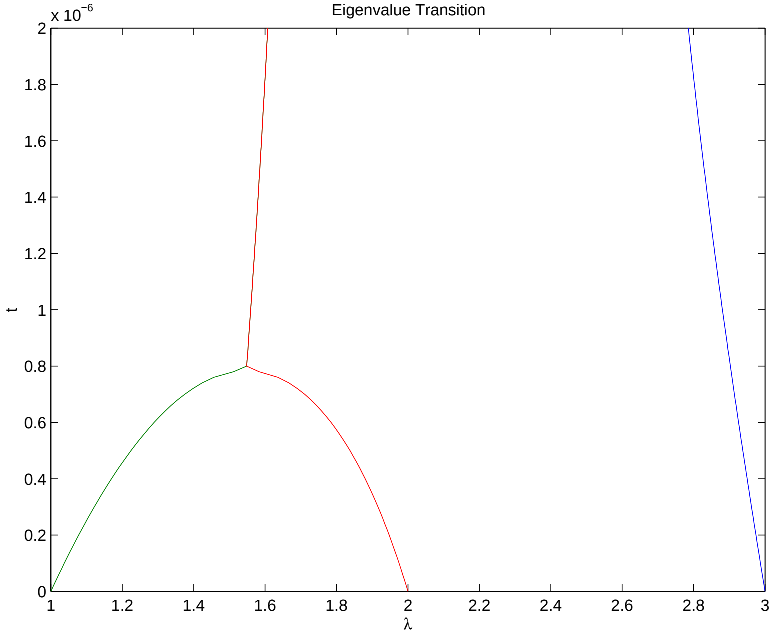
```
[                        1/3                                   ]
[                  1/3 %1    - 3 %2 + 2 + 1/3 t                ]
[                                                              ]
[        1/3                              1/2        1/3       ]
[- 1/6 %1    + 3/2 %2 + 2 + 1/3 t + 1/2 i 3    (1/3 %1    + 3 %2)]
[                                                              ]
[        1/3                              1/2        1/3       ]
[- 1/6 %1    + 3/2 %2 + 2 + 1/3 t - 1/2 i 3    (1/3 %1    + 3 %2)]

                           2    3
%1 := 3189393 t - 2216286 t  + t  + 3 (-3 + 4432572 t
                      2                       3
     - 1052829647418 t + 358392752910068940 t
                     4 1/2
     - 181922388795 t )

                                  2
      - 1/3 + 492508/3 t - 1/9 t
%2 := ---------------------------
                    1/3
                  %1
```

Next, the symbolic representations of the three eigenvalues are evaluated at many values of `t`

```
tvals = (2:-.02:0)' * 1.e-6;
r = size(tvals,1);
c = size(e,1);
lambda = zeros(r,c);
for k = 1:c
   lambda(:,k) = double(subs(e(k),t,tvals));
end
```

```
plot(lambda,tvals)
xlabel('\lambda'); ylabel('t');
title('Eigenvalue Transition')
```

to produce a plot of their trajectories.



Above `t = 0.8e`$^{-6}$, the graphs of two of the eigenvalues intersect, while below `t` = 0.8e^{-6}, two real roots become a complex conjugate pair. What is the precise value of `t` that marks this transition? Let τ denote this value of `t`.

One way to find τ is based on the fact that, at a double root, both the function and its derivative must vanish. This results in two polynomial equations to be solved for two unknowns. The statement

```
sol = solve(p,diff(p,'x'))
```

solves the pair of algebraic equations `p = 0` and `dp/dx = 0` and produces

```
sol =
    t: [4x1 sym]
    x: [4x1 sym]
```

Find τ now by

```
tau = double(sol.t(2))
```

which reveals that the second element of `sol.t` is the desired value of τ.

```
format short
tau =
   7.8379e-07
```

Therefore, the second element of `sol.x`

```
sigma = double(sol.x(2))
```

is the double eigenvalue

```
sigma =
   1.5476
```

Let's verify that this value of τ does indeed produce a double eigenvalue at $\sigma = 1.5476$. To achieve this, substitute τ for t in the perturbed matrix $A(t) = A + tE$ and find the eigenvalues of $A(t)$. That is,

```
e = eig(double(subs(A,t,tau)))
```

```
e =

   1.5476
   1.5476
   2.9047
```

confirms that $\sigma = 1.5476$ is a double eigenvalue of $A(t)$ for t = 7.8379e-07.

Solving Equations

Solving Algebraic Equations

If `S` is a symbolic expression,

```
solve(S)
```

attempts to find values of the symbolic variable in `S` (as determined by `findsym`) for which `S` is zero. For example,

```
syms a b c x
S = a*x^2 + b*x + c;
solve(S)
```

uses the familiar quadratic formula to produce

```
ans =
[1/2/a*(-b+(b^2-4*a*c)^(1/2))]
[1/2/a*(-b-(b^2-4*a*c)^(1/2))]
```

This is a symbolic vector whose elements are the two solutions.

If you want to solve for a specific variable, you must specify that variable as an additional argument. For example, if you want to solve `S` for `b`, use the command

```
b = solve(S,b)
```

which returns

```
b =
-(a*x^2+c)/x
```

Note that these examples assume equations of the form $f(x) = 0$. If you need to solve equations of the form $f(x) = q(x)$, you must use quoted strings. In particular, the command

```
s = solve('cos(2*x)+sin(x)=1')
```

returns a vector with four solutions.

```
s =
[      0]
[     pi]
[ 1/6*pi]
[ 5/6*pi]
```

Several Algebraic Equations

Now let's look at systems of equations. Suppose we have the system

$$x^2 y^2 = 0$$

$$x - \frac{y}{2} = \alpha$$

and we want to solve for x and y. First create the necessary symbolic objects.

```
syms x y alpha
```

There are several ways to address the output of solve. One is to use a two-output call

```
[x,y] = solve(x^2*y^2, x-y/2-alpha)
```

which returns

```
x =
[     0]
[     0]
[ alpha]
[ alpha]

y =
[ -2*alpha]
[ -2*alpha]
[        0]
[        0]
```

Consequently, the solution vector

```
v = [x, y]
```

appears to have redundant components. This is due to the first equation $x^2y^2 = 0$, which has two solutions in *x* and *y*: $x = \pm 0$, $y = \pm 0$. Changing the equations to

```
eqs1 = 'x^2*y^2=1, x-y/2-alpha'
[x,y] = solve(eqs1)
```

produces four distinct solutions.

```
x =
[ 1/2*alpha+1/2*(alpha^2+2)^(1/2)]
[ 1/2*alpha-1/2*(alpha^2+2)^(1/2)]
[ 1/2*alpha+1/2*(alpha^2-2)^(1/2)]
[ 1/2*alpha-1/2*(alpha^2-2)^(1/2)]


y =
[ -alpha+(alpha^2+2)^(1/2)]
[ -alpha-(alpha^2+2)^(1/2)]
[ -alpha+(alpha^2-2)^(1/2)]
[ -alpha-(alpha^2-2)^(1/2)]
```

Since we did not specify the dependent variables, `solve` uses `findsym` to determine the variables.

This way of assigning output from `solve` is quite successful for “small” systems. Plainly, if we had, say, a 10-by-10 system of equations, typing

```
[x1,x2,x3,x4,x5,x6,x7,x8,x9,x10] = solve(...)
```

is both awkward and time consuming. To circumvent this difficulty, `solve` can return a structure whose fields are the solutions. In particular, consider the system `u^2-v^2 = a^2`, `u + v = 1`, `a^2-2*a = 3`. The command

```
S = solve('u^2-v^2 = a^2','u + v = 1','a^2-2*a = 3')
```

returns

```
S =
    a: [2x1 sym]
    u: [2x1 sym]
    v: [2x1 sym]
```

The solutions for a reside in the “a-field” of S. That is,

```
S.a
```

produces

```
ans =
[ -1]
[  3]
```

Similar comments apply to the solutions for u and v. The structure S can now be manipulated by field and index to access a particular portion of the solution. For example, if we want to examine the second solution, we can use the following statement

```
s2 = [S.a(2), S.u(2), S.v(2)]
```

to extract the second component of each field.

```
s2 =
[  3,  5, -4]
```

The following statement

```
M = [S.a, S.u, S.v]
```

creates the solution matrix M

```
M =
[ -1,  1,  0]
[  3,  5, -4]
```

whose rows comprise the distinct solutions of the system.

Linear systems of simultaneous equations can also be solved using matrix division. For example,

```
clear u v x y
syms u v x y
S = solve(x+2*y-u, 4*x+5*y-v);
sol = [S.x;S.y]
```

and

```
A = [1 2; 4 5];
b = [u; v];
z = A\b
```

result in

```
sol =

[ -5/3*u+2/3*v]
[  4/3*u-1/3*v]

z =
[ -5/3*u+2/3*v]
[  4/3*u-1/3*v]
```

Thus `s` and `z` produce the same solution, although the results are assigned to different variables.

Single Differential Equation

The function `dsolve` computes symbolic solutions to ordinary differential equations. The equations are specified by symbolic expressions containing the letter `D` to denote differentiation. The symbols `D2`, `D3`, ... `DN`, correspond to the second, third, ..., Nth derivative, respectively. Thus, `D2y` is the Symbolic Math Toolbox equivalent of d^2y/dt^2. The dependent variables are those preceded by `D` and the default independent variable is `t`. Note that names of symbolic variables should not contain `D`. The independent variable can be changed from `t` to some other symbolic variable by including that variable as the last input argument.

Initial conditions can be specified by additional equations. If initial conditions are not specified, the solutions contain constants of integration, `C1`, `C2`, etc.

The output from `dsolve` parallels the output from `solve`. That is, you can call `dsolve` with the number of output variables equal to the number of dependent variables or place the output in a structure whose fields contain the solutions of the differential equations.

Example 1

The following call to `dsolve`

```
dsolve('Dy=1+y^2')
```

uses y as the dependent variable and t as the default independent variable. The output of this command is

```
ans =
tan(t+C1)
```

To specify an initial condition, use

```
y = dsolve('Dy=1+y^2','y(0)=1')
```

This produces

```
y =
tan(t+1/4*pi)
```

Notice that y is in the MATLAB workspace, but the independent variable t is not. Thus, the command diff(y,t) returns an error. To place t in the workspace, type syms t.

Example 2

Nonlinear equations may have multiple solutions, even when initial conditions are given.

```
x = dsolve('(Dx)^2+x^2=1','x(0)=0')
```

results in

```
x =
[-sin(t)]
[ sin(t)]
```

Example 3

Here is a second order differential equation with two initial conditions. The commands

```
y = dsolve('D2y=cos(2*x)-y','y(0)=1','Dy(0)=0', 'x')
simplify(y)
```

produce

```
y =
-2/3*cos(x)^2+1/3+4/3*cos(x)
```

The key issues in this example are the order of the equation and the initial conditions. To solve the ordinary differential equation

$$\frac{d^3u}{dx^3} = u$$

$$u(0) = 1, u'(0) = -1, u''(0) = \pi$$

simply type

```
u = dsolve('D3u=u','u(0)=1','Du(0)=-1','D2u(0) = pi','x')
```

Use `D3u` to represent d^3u/dx^3 and `D2u(0)` for $u''(0)$.

Several Differential Equations

The function `dsolve` can also handle several ordinary differential equations in several variables, with or without initial conditions. For example, here is a pair of linear, first order equations.

```
S = dsolve('Df = 3*f+4*g', 'Dg = -4*f+3*g')
```

The computed solutions are returned in the structure `S`. You can determine the values of `f` and `g` by typing

```
f = S.f
f =
exp(3*t)*(cos(4*t)*C1+sin(4*t)*C2)

g = S.g
g =
-exp(3*t)*(sin(4*t)*C1-cos(4*t)*C2)
```

If you prefer to recover `f` and `g` directly as well as include initial conditions, type

```
[f,g] = dsolve('Df=3*f+4*g, Dg =-4*f+3*g', 'f(0) = 0, g(0) = 1')

f =
exp(3*t)*sin(4*t)

g =
exp(3*t)*cos(4*t)
```

This table details some examples and Symbolic Math Toolbox syntax. Note that the final entry in the table is the Airy differential equation whose solution is referred to as the Airy function.

Differential Equation	MATLAB Command
$\frac{dy}{dt} + 4y(t) = e^{-t}$ $y(0) = 1$	`y = dsolve('Dy+4*y = exp(-t)',` `'y(0) = 1')`
$\frac{d^2y}{dx^2} + 4y(x) = e^{-2x}$ $y(0) = 0, y(\pi) = 0$	`y = dsolve('D2y+4*y = exp(-2*x)',` `'y(0)=0', 'y(pi) = 0', 'x')`

$\frac{d^2y}{dx^2} = xy(x)$
 	
$y(0) = 0, y(3) = \frac{1}{\pi}K_{\frac{1}{3}}(2\sqrt{3})$
 	
(The Airy Equation)
	`y = dsolve('D2y = x*y','y(0) = 0',`
	`'y(3) = besselk(1/3, 2*sqrt(3))/pi',`
	`'x')`

The Airy function plays an important role in the mathematical modeling of the dispersion of water waves.

A

MATLAB Quick Reference

Introduction

This appendix lists the MATLAB functions as they are grouped in Help by subject. Each table contains the function names and brief descriptions. For complete information about any of these functions, refer to Help and either:

- Select the function from the MATLAB Function Reference (Functions by Category or Alphabetical List of Functions), or
- From the **Search** tab in the Help Navigator, select **Function Name** as **Search type**, type the function name in the **Search for** field, and click **Go**.

Note If you are viewing this book from Help, you can click on any function name and jump directly to the corresponding MATLAB function page.

General Purpose Commands

This set of functions lets you start and stop MATLAB, work with files and the operating system, control the command window, and manage the environment, variables, and the workspace.

Managing Commands and Functions	
addpath	Add directories to MATLAB's search path
doc	Display HTML documentation in Help browser
docopt	Display location of help file directory for UNIX platforms
genpath	Generate a path string
help	Display M-file help for MATLAB functions in the Command Window
helpbrowser	Display Help browser for access to all MathWorks online help
helpdesk	Display Help browser
helpwin	Display M-file help and provide access to M-file help for all functions
lasterr	Last error message
lastwarn	Last warning message
license	Show MATLAB license number
lookfor	Search for specified keyword in M-file help entries
partialpath	Partial pathname
path	Control MATLAB's directory search path
pathtool	Open the GUI for viewing and modifying MATLAB's path
profile	Start the M-file profiler, a utility for debugging and optimizing code
profreport	Generate a profile report

Managing Commands and Functions (Continued)	
rehash	Refresh function and file system caches
rmpath	Remove directories from MATLAB's search path
support	Open MathWorks Technical Support Web page
type	List file
ver	Display version information for MATLAB, Simulink, and toolboxes
version	Get MATLAB version number
web	Point Help browser or Web browser at file or Web site
what	List MATLAB-specific files in current directory
whatsnew	Display README files for MATLAB and toolboxes
which	Locate functions and files

Managing Variables and the Workspace	
clear	Remove items from the workspace
disp	Display text or array
length	Length of vector
load	Retrieve variables from disk
memory	Help for memory limitations
mlock	Prevent M-file clearing
munlock	Allow M-file clearing
openvar	Open workspace variable in Array Editor for graphical editing
pack	Consolidate workspace memory
save	Save workspace variables on disk
saveas	Save figure or model using specified format

Managing Variables and the Workspace (Continued)	
`size`	Array dimensions
`who, whos`	List the variables in the workspace
`workspace`	Display the Workspace browser, a GUI for managing the workspace

Controlling the Command Window	
`clc`	Clear Command Window
`echo`	Echo M-files during execution
`format`	Control the display format for output
`home`	Move cursor to upper left corner of Command Window
`more`	Control paged output for the command window

Working with Files and the Operating Environment	
`beep`	Produce a beep sound
`cd`	Change working directory
`checkin`	Check file into source control system
`checkout`	Check file out of source control system
`cmopts`	Get name of source control system, and PVCS project filename
`copyfile`	Copy file
`customverctrl`	Allow custom source control system
`delete`	Delete files and graphics objects
`diary`	Save session to a disk file
`dir`	Display a directory listing
`edit`	Edit an M-file
`fileparts`	Get filename parts

Working with Files and the Operating Environment (Continued)	
`filebrowser`	Display Current Directory browser, for viewing files
`fullfile`	Build full filename from parts
`info`	Display contact information or toolbox Readme files
`inmem`	Functions in memory
`ls`	List directory on UNIX
`matlabroot`	Get root directory of MATLAB installation
`mkdir`	Make new directory
`open`	Open files based on extension
`pwd`	Display current directory
`tempdir`	Return the name of the system's temporary directory
`tempname`	Unique name for temporary file
`undocheckout`	Undo previous checkout from source control system
`unix`	Execute a UNIX command and return the result
`!`	Execute operating system command

Starting and Quitting MATLAB	
`finish`	MATLAB termination M-file
`exit`	Terminate MATLAB
`matlab`	Start MATLAB (UNIX systems only)
`matlabrc`	MATLAB startup M-file
`quit`	Terminate MATLAB
`startup`	MATLAB startup M-file

Operators and Special Characters

These are the actual operators you use to enter and manipulate data, for example, matrix multiplication, array multiplication, and line continuation.

Operators and Special Characters	
+	Plus
-	Minus
*	Matrix multiplication
.*	Array multiplication
^	Matrix power
.^	Array power
kron	Kronecker tensor product
\	Backslash or left division
/	Slash or right division
./ and .\	Array division, right and left
:	Colon
( )	Parentheses
[ ]	Brackets
{}	Curly braces
.	Decimal point
...	Continuation
,	Comma
;	Semicolon
%	Comment
!	Exclamation point
'	Transpose and quote
.'	Nonconjugated transpose
=	Assignment
==	Equality
< >	Relational operators
&	Logical AND
|	Logical OR
~	Logical NOT
xor	Logical EXCLUSIVE OR

Logical Functions

This set of functions performs logical operations such as checking if a file or variable exists and testing if all elements in an array are nonzero. "Operators and Special Characters" contains other operators that perform logical operations.

Logical Functions	
all	Test to determine if all elements are nonzero
any	Test for any nonzeros
exist	Check if a variable or file exists
find	Find indices and values of nonzero elements
is*	Detect state
isa	Detect an object of a given class
iskeyword	Test if string is a MATLAB keyword
isvarname	Test if string is a valid variable name
logical	Convert numeric values to logical
mislocked	True if M-file cannot be cleared

Language Constructs and Debugging

These functions let you work with MATLAB as a programming language. For example, you can control program flow, define global variables, perform interactive input, and debug your code.

MATLAB as a Programming Language	
builtin	Execute builtin function from overloaded method
eval	Interpret strings containing MATLAB expressions
evalc	Evaluate MATLAB expression with capture

MATLAB as a Programming Language (Continued)	
`evalin`	Evaluate expression in workspace
`feval`	Function evaluation
`function`	Function M-files
`global`	Define global variables
`nargchk`	Check number of input arguments
`persistent`	Define persistent variable
`script`	Script M-files

Control Flow	
`break`	Terminate execution of `for` loop or `while` loop
`case`	Case switch
`catch`	Begin catch block
`continue`	Pass control to the next iteration of `for` or `while` loop
`else`	Conditionally execute statements
`elseif`	Conditionally execute statements
`end`	Terminate `for`, `while`, `switch`, `try`, and `if` statements or indicate last index
`error`	Display error messages
`for`	Repeat statements a specific number of times
`if`	Conditionally execute statements
`otherwise`	Default part of `switch` statement
`return`	Return to the invoking function
`switch`	Switch among several cases based on expression
`try`	Begin try block

Control Flow (Continued)	
`warning`	Display warning message
`while`	Repeat statements an indefinite number of times

Interactive Input	
`input`	Request user input
`keyboard`	Invoke the keyboard in an M-file
`menu`	Generate a menu of choices for user input
`pause`	Halt execution temporarily

Object-Oriented Programming	
`class`	Create object or return class of object
`double`	Convert to double precision
`inferiorto`	Inferior class relationship
`inline`	Construct an inline object
`int8, int16, int32`	Convert to signed integer
`isa`	Detect an object of a given class
`loadobj`	Extends the `load` function for user objects
`saveobj`	Save filter for objects
`single`	Convert to single precision
`superiorto`	Superior class relationship
`uint8, uint16, uint32`	Convert to unsigned integer

Debugging	
`dbclear`	Clear breakpoints
`dbcont`	Resume execution
`dbdown`	Change local workspace context
`dbmex`	Enable MEX-file debugging
`dbquit`	Quit debug mode
`dbstack`	Display function call stack
`dbstatus`	List all breakpoints

Debugging (Continued)	
dbstep	Execute one or more lines from a breakpoint
dbstop	Set breakpoints in an M-file function
dbtype	List M-file with line numbers
dbup	Change local workspace context

Function Handles	
function_handle	MATLAB data type that is a handle to a function
functions	Return information about a function handle
func2str	Constructs a function name string from a function handle
str2func	Constructs a function handle from a function name string

Elementary Matrices and Matrix Manipulation

Using these functions you can manipulate matrices, and access time, date, special variables, and constants, functions.

Elementary Matrices and Arrays	
blkdiag	Construct a block diagonal matrix from input arguments
eye	Identity matrix
linspace	Generate linearly spaced vectors
logspace	Generate logarithmically spaced vectors
numel	Number of elements in a matrix or cell array
ones	Create an array of all ones
rand	Uniformly distributed random numbers and arrays
randn	Normally distributed random numbers and arrays

Elementary Matrices and Arrays (Continued)	
zeros	Create an array of all zeros
: (colon)	Regularly spaced vector

Special Variables and Constants	
ans	The most recent answer
computer	Identify the computer on which MATLAB is running
eps	Floating-point relative accuracy
i	Imaginary unit
Inf	Infinity
inputname	Input argument name
j	Imaginary unit
NaN	Not-a-Number
nargin, nargout	Number of function arguments
nargoutchk	Validate number of output arguments
pi	Ratio of a circle's circumference to its diameter
realmax	Largest positive floating-point number
realmin	Smallest positive floating-point number
varargin, varargout	Pass or return variable numbers of arguments

Time and Dates	
calendar	Calendar
clock	Current time as a date vector
cputime	Elapsed CPU time
date	Current date string
datenum	Serial date number
datestr	Date string format
datevec	Date components
eomday	End of month
etime	Elapsed time
now	Current date and time

Time and Dates (Continued)	
`tic, toc`	Stopwatch timer
`weekday`	Day of the week

Matrix Manipulation	
`cat`	Concatenate arrays
`diag`	Diagonal matrices and diagonals of a matrix
`fliplr`	Flip matrices left-right
`flipud`	Flip matrices up-down
`repmat`	Replicate and tile an array
`reshape`	Reshape array
`rot90`	Rotate matrix 90 degrees
`tril`	Lower triangular part of a matrix
`triu`	Upper triangular part of a matrix
`: (colon)`	Index into array, rearrange array

Vector Functions	
`cross`	Vector cross product
`dot`	Vector dot product
`intersect`	Set intersection of two vectors
`ismember`	Detect members of a set
`setdiff`	Return the set difference of two vectors
`setxor`	Set exclusive or of two vectors
`union`	Set union of two vectors
`unique`	Unique elements of a vector

Specialized Matrices

These functions let you work with matrices such as Hadamard, Hankel, Hilbert, and magic squares.

Specialized Matrices	
`compan`	Companion matrix
`gallery`	Test matrices
`hadamard`	Hadamard matrix
`hankel`	Hankel matrix
`hilb`	Hilbert matrix
`invhilb`	Inverse of the Hilbert matrix
`magic`	Magic square
`pascal`	Pascal matrix
`toeplitz`	Toeplitz matrix
`wilkinson`	Wilkinson's eigenvalue test matrix

Elementary Math Functions

These are many of the standard mathematical functions such as trigonometric, hyperbolic, logarithmic, and complex number manipulation.

Elementary Math Functions	
`abs`	Absolute value and complex magnitude
`acos, acosh`	Inverse cosine and inverse hyperbolic cosine
`acot, acoth`	Inverse cotangent and inverse hyperbolic cotangent
`acsc, acsch`	Inverse cosecant and inverse hyperbolic cosecant
`angle`	Phase angle
`asec, asech`	Inverse secant and inverse hyperbolic secant
`asin, asinh`	Inverse sine and inverse hyperbolic sine

Elementary Math Functions (Continued)	
atan, atanh	Inverse tangent and inverse hyperbolic tangent
atan2	Four-quadrant inverse tangent
ceil	Round toward infinity
complex	Construct complex data from real and imaginary components
conj	Complex conjugate
cos, cosh	Cosine and hyperbolic cosine
cot, coth	Cotangent and hyperbolic cotangent
csc, csch	Cosecant and hyperbolic cosecant
exp	Exponential
fix	Round towards zero
floor	Round towards minus infinity
gcd	Greatest common divisor
imag	Imaginary part of a complex number
lcm	Least common multiple
log	Natural logarithm
log2	Base 2 logarithm and dissect floating-point numbers into exponent and mantissa
log10	Common (base 10) logarithm
mod	Modulus (signed remainder after division)
nchoosek	Binomial coefficient or all combinations
real	Real part of complex number
rem	Remainder after division
round	Round to nearest integer
sec, sech	Secant and hyperbolic secant
sign	Signum function
sin, sinh	Sine and hyperbolic sine
sqrt	Square root
tan, tanh	Tangent and hyperbolic tangent

Specialized Math Functions

This set of functions includes Bessel, elliptic, gamma, factorial, and others.

Specialized Math Functions	
airy	Airy functions
besselh	Bessel functions of the third kind (Hankel functions)
besseli, besselk	Modified Bessel functions
besselj, bessely	Bessel functions
beta, betainc, betaln	beta, betainc, betaln
ellipj	Jacobi elliptic functions
ellipke	Complete elliptic integrals of the first and second kind
erf, erfc, erfcx, erfinv	Error functions
expint	Exponential integral
factorial	Factorial function
gamma, gammainc, gammaln	Gamma functions
legendre	Associated Legendre functions
pow2	Base 2 power and scale floating-point numbers
rat, rats	Rational fraction approximation

Coordinate System Conversion

Using these functions you can transform Cartesian coordinates to polar, cylindrical, or spherical, and vice versa.

Coordinate System Conversion	
cart2pol	Transform Cartesian coordinates to polar or cylindrical
cart2sph	Transform Cartesian coordinates to spherical
pol2cart	Transform polar or cylindrical coordinates to Cartesian
sph2cart	Transform spherical coordinates to Cartesian

Matrix Functions - Numerical Linear Algebra

These functions let you perform matrix analysis including matrix determinant, rank, reduced row echelon form, eigenvalues, and inverses.

Matrix Analysis	
cond	Condition number with respect to inversion
condeig	Condition number with respect to eigenvalues
det	Matrix determinant
norm	Vector and matrix norms
null	Null space of a matrix
orth	Range space of a matrix
rank	Rank of a matrix
rcond	Matrix reciprocal condition number estimate
rref, rrefmovie	Reduced row echelon form
subspace	Angle between two subspaces
trace	Sum of diagonal elements

Linear Equations	
chol	Cholesky factorization
inv	Matrix inverse
lscov	Least squares solution in the presence of known covariance
lu	LU matrix factorization
lsqnonneg	Nonnegative least squares
minres	Minimum Residual Method
pinv	Moore-Penrose pseudoinverse of a matrix
qr	Orthogonal-triangular decomposition
symmlq	Symmetric LQ method

Eigenvalues and Singular Values	
balance	Improve accuracy of computed eigenvalues
cdf2rdf	Convert complex diagonal form to real block diagonal form
eig	Eigenvalues and eigenvectors
gsvd	Generalized singular value decomposition
hess	Hessenberg form of a matrix
poly	Polynomial with specified roots
qz	QZ factorization for generalized eigenvalues
rsf2csf	Convert real Schur form to complex Schur form
schur	Schur decomposition
svd	Singular value decomposition

Matrix Functions	
expm	Matrix exponential
funm	Evaluate general matrix function
logm	Matrix logarithm
sqrtm	Matrix square root

Low Level Functions	
qrdelete	Delete column from QR factorization
qrinsert	Insert column in QR factorization

Data Analysis and Fourier Transform Functions

Using the data analysis functions, you can find permutations, prime numbers, mean, median, variance, correlation, and perform convolutions and other standard array manipulations. A set of vector functions lets you operate on vectors to find cross product, union, and other standard vector manipulations. The Fourier transform functions let you perform discrete Fourier transformations in one or more dimensions and their inverses.

Basic Operations	
cumprod	Cumulative product
cumsum	Cumulative sum
cumtrapz	Cumulative trapezoidal numerical integration
factor	Prime factors
inpolygon	Detect points inside a polygonal region
max	Maximum elements of an array
mean	Average or mean value of arrays
median	Median value of arrays
min	Minimum elements of an array
perms	All possible permutations
polyarea	Area of polygon
primes	Generate list of prime numbers
prod	Product of array elements
rectint	Rectangle intersection area
sort	Sort elements in ascending order
sortrows	Sort rows in ascending order

Basic Operations (Continued)	
std	Standard deviation
sum	Sum of array elements
trapz	Trapezoidal numerical integration
var	Variance

Finite Differences	
del2	Discrete Laplacian
diff	Differences and approximate derivatives
gradient	Numerical gradient

Correlation	
corrcoef	Correlation coefficients
cov	Covariance matrix

Filtering and Convolution	
conv	Convolution and polynomial multiplication
conv2	Two-dimensional convolution
deconv	Deconvolution and polynomial division
filter	Filter data with an infinite impulse response (IIR) or finite impulse response (FIR) filter
filter2	Two-dimensional digital filtering

Fourier Transforms	
abs	Absolute value and complex magnitude
angle	Phase angle
cplxpair	Sort complex numbers into complex conjugate pairs
fft	One-dimensional fast Fourier transform

Fourier Transforms (Continued)	
fft2	Two-dimensional fast Fourier transform
fftshift	Shift DC component of fast Fourier transform to center of spectrum
ifft	Inverse one-dimensional fast Fourier transform
ifft2	Inverse two-dimensional fast Fourier transform
ifftn	Inverse multidimensional fast Fourier transform
ifftshift	Inverse FFT shift
nextpow2	Next power of two
unwrap	Correct phase angles

Vector Functions	
cross	Vector cross product
intersect	Set intersection of two vectors
ismember	Detect members of a set
setdiff	Return the set difference of two vector
setxor	Set exclusive or of two vectors
union	Set union of two vectors
unique	Unique elements of a vector

Polynomial and Interpolation Functions

These functions let you operate on polynomials such as multiply, divide, find derivatives, and evaluate. The data interpolation functions let you perform interpolation in one, two, three, and higher dimensions.

Polynomials	
conv	Convolution and polynomial multiplication
deconv	Deconvolution and polynomial division
poly	Polynomial with specified roots
polyder	Polynomial derivative
polyeig	Polynomial eigenvalue problem
polyfit	Polynomial curve fitting
polyint	Analytic polynomial integration
polyval	Polynomial evaluation
polyvalm	Matrix polynomial evaluation
residue	Convert between partial fraction expansion and polynomial coefficients
roots	Polynomial roots

Data Interpolation	
convhull	Convex hull
convhulln	Multidimensional convex hull
delaunay	Delaunay triangulation
delaunay3	3-D Delaunay tessellation
delaunayn	Multidimensional Delaunay tessellation
dsearch	Search for nearest point
dsearchn	Multidimensional closest point search
griddata	Data gridding
griddata3	Data gridding and hypersurface fitting for three-dimensional data
griddatan	Data gridding and hypersurface fitting (dimension >= 2)
interp1	One-dimensional data interpolation (table lookup)
interp2	Two-dimensional data interpolation (table lookup)

Data Interpolation (Continued)	
interp3	Three-dimensional data interpolation (table lookup)
interpft	One-dimensional interpolation using the FFT method
interpn	Multidimensional data interpolation (table lookup)
meshgrid	Generate X and Y matrices for three-dimensional plots
ndgrid	Generate arrays for multidimensional functions and interpolation
pchip	Piecewise Cubic Hermite Interpolating Polynomial (PCHIP)
ppval	Piecewise polynomial evaluation
spline	Cubic spline interpolation
tsearch	Search for enclosing Delaunay triangle
tsearchn	Multidimensional closest simplex search
voronoi	Voronoi diagram
voronoin	Multidimensional Voronoi diagrams

Function Functions - Nonlinear Numerical Methods

Using these functions you can solve differential equations, perform numerical evaluation of integrals, and optimize functions.

Function Functions - Nonlinear Numerical Methods	
bvp4c	Solve two-point boundary value problems (BVPs) for ordinary differential equations (ODEs)
bvpget	Extract parameters from BVP options structure
bvpinit	Form the initial guess for bvp4c

Function Functions - Nonlinear Numerical Methods (Continued)	
bvpset	Create/alter BVP options structure
bvpval	Evaluate the solution computed by bvp4c
dblquad	Numerical evaluation of double integrals
fminbnd	Minimize a function of one variable
fminsearch	Minimize a function of several variables
fzero	Find zero of a function of one variable
ode45, ode23, ode113, ode15s, ode23s, ode23t, ode23tb	Solve initial value problems for ODEs
odeget	Extract parameters from ODE options structure
odeset	Create/alter ODE options structure
optimget	Get optimization options structure parameter values
optimset	Create or edit optimization options parameter structure
pdepe	Solve initial-boundary value problems
pdeval	Evaluate the solution computed by pdepe
quad	Numerical evaluation of integrals, adaptive Simpson quadrature
quadl	Numerical evaluation of integrals, adaptive Lobatto quadrature
vectorize	Vectorize expression

Sparse Matrix Functions

These functions allow you to operate on a special type of matrix, sparse. Using these functions you can convert full to sparse, visualize, and operate on these matrices.

Elementary Sparse Matrices	
spdiags	Extract and create sparse band and diagonal matrices
speye	Sparse identity matrix
sprand	Sparse uniformly distributed random matrix
sprandn	Sparse normally distributed random matrix
sprandsym	Sparse symmetric random matrix

Full to Sparse Conversion	
find	Find indices and values of nonzero elements
full	Convert sparse matrix to full matrix
sparse	Create sparse matrix
spconvert	Import matrix from sparse matrix external format

Working with Nonzero Entries of Sparse Matrices	
nnz	Number of nonzero matrix elements
nonzeros	Nonzero matrix elements
nzmax	Amount of storage allocated for nonzero matrix elements
spalloc	Allocate space for sparse matrix
spfun	Apply function to nonzero sparse matrix elements
spones	Replace nonzero sparse matrix elements with ones

Visualizing Sparse Matrices	
spy	Visualize sparsity pattern

Reordering Algorithms	
colamd	Column approximate minimum degree permutation
colmmd	Sparse column minimum degree permutation
colperm	Sparse column permutation based on nonzero count
dmperm	Dulmage-Mendelsohn decomposition
randperm	Random permutation
symamd	Symmetric approximate minimum degree permutation
symmmd	Sparse symmetric minimum degree ordering
symrcm	Sparse reverse Cuthill-McKee ordering

Norm, Condition Number, and Rank	
condest	1-norm matrix condition number estimate
normest	2-norm estimate

Sparse Systems of Linear Equations	
bicg	BiConjugate Gradients method
bicgstab	BiConjugate Gradients Stabilized method
cgs	Conjugate Gradients Squared method
cholinc	Sparse Incomplete Cholesky and Cholesky-Infinity factorizations
cholupdate	Rank 1 update to Cholesky factorization
gmres	Generalized Minimum Residual method (with restarts)

Sparse Systems of Linear Equations (Continued)	
`lsqr`	LSQR implementation of Conjugate Gradients on the normal equations
`luinc`	Incomplete LU matrix factorizations
`pcg`	Preconditioned Conjugate Gradients method
`qmr`	Quasi-Minimal Residual method
`qr`	Orthogonal-triangular decomposition
`qrdelete`	Delete column from QR factorization
`qrinsert`	Insert column in QR factorization
`qrupdate`	Rank 1 update to QR factorization

Sparse Eigenvalues and Singular Values	
`eigs`	Find eigenvalues and eigenvectors
`svds`	Find singular values

Miscellaneous	
`spparms`	Set parameters for sparse matrix routines

Sound Processing Functions

The sound processing functions let you convert signals, and read and write `.au` and `.wav` sound files.

General Sound Functions	
`lin2mu`	Convert linear audio signal to mu-law
`mu2lin`	Convert mu-law audio signal to linear
`sound`	Convert vector into sound
`soundsc`	Scale data and play as sound

SPARCstation-Specific Sound Functions	
`auread`	Read NeXT/SUN (.au) sound file
`auwrite`	Write NeXT/SUN (.au) sound file

.WAV Sound Functions	
`wavplay`	Play recorded sound on a PC-based audio output device
`wavread`	Read Microsoft WAVE (.wav) sound file
`wavrecord`	Record sound using a PC-based audio input device
`wavwrite`	Write Microsoft WAVE (.wav) sound file

Character String Functions

This set of functions lets you manipulate strings such as comparison, concatenation, search, and conversion.

General	
abs	Absolute value and complex magnitude
eval	Interpret strings containing MATLAB expressions
real	Real part of complex number
strings	MATLAB string handling

String to Function Handle Conversion	
func2str	Constructs a function name string from a function handle
str2func	Constructs a function handle from a function name string

String Manipulation	
deblank	Strip trailing blanks from the end of a string
findstr	Find one string within another
lower	Convert string to lower case
strcat	String concatenation
strcmp	Compare strings
strcmpi	Compare strings ignoring case
strjust	Justify a character array
strmatch	Find possible matches for a string
strncmp	Compare the first `n` characters of two strings
strncmpi	Compare the first n characters of strings, ignoring case
strrep	String search and replace
strtok	First token in string
strvcat	Vertical concatenation of strings

String Manipulation (Continued)	
symvar	Determine symbolic variables in an expression
texlabel	Produce the TeX format from a character string
upper	Convert string to upper case

String to Number Conversion	
char	Create character array (string)
int2str	Integer to string conversion
mat2str	Convert a matrix into a string
num2str	Number to string conversion
sprintf	Write formatted data to a string
sscanf	Read string under format control
str2double	Convert string to double-precision value
str2mat	String to matrix conversion
str2num	String to number conversion

Radix Conversion	
bin2dec	Binary to decimal number conversion
dec2bin	Decimal to binary number conversion
dec2hex	Decimal to hexadecimal number conversion
hex2dec	Hexadecimal to decimal number conversion
hex2num	Hexadecimal to double number conversion

File I/O Functions

The file I/O functions allow you to open and close files, read and write formatted and unformatted data, operate on files, and perform other specialized file I/O such as reading and writing images and spreadsheets.

File Opening and Closing	
`fclose`	Close one or more open files
`fopen`	Open a file or obtain information about open files

Unformatted I/O	
`fread`	Read binary data from file
`fwrite`	Write binary data to a file

Formatted I/O	
`fgetl`	Return the next line of a file as a string without line terminator(s)
`fgets`	Return the next line of a file as a string with line terminator(s)
`fprintf`	Write formatted data to file
`fscanf`	Read formatted data from file

File Positioning	
`feof`	Test for end-of-file
`ferror`	Query MATLAB about errors in file input or output
`frewind`	Rewind an open file
`fseek`	Set file position indicator
`ftell`	Get file position indicator

String Conversion	
`sprintf`	Write formatted data to a string
`sscanf`	Read string under format control

Specialized File I/O	
`dlmread`	Read an ASCII delimited file into a matrix
`dlmwrite`	Write a matrix to an ASCII delimited file
`hdf`	HDF interface
`imfinfo`	Return information about a graphics file
`imread`	Read image from graphics file
`imwrite`	Write an image to a graphics file
`strread`	Read formatted data from a string
`textread`	Read formatted data from text file
`wk1read`	Read a Lotus123 `WK1` spreadsheet file into a matrix
`wk1write`	Write a matrix to a Lotus123 `WK1` spreadsheet file

Bitwise Functions

These functions let you operate at the bit level such as shifting and complementing.

Bitwise Functions	
`bitand`	Bit-wise AND
`bitcmp`	Complement bits
`bitor`	Bit-wise OR
`bitmax`	Maximum floating-point integer
`bitset`	Set bit
`bitshift`	Bit-wise shift
`bitget`	Get bit
`bitxor`	Bit-wise XOR

Structure Functions

Structures are arrays whose elements can hold any MATLAB data type such as text, numeric arrays, or other structures. You access structure elements by name. Use the structure functions to create and operate on this array type.

Structure Functions	
`deal`	Deal inputs to outputs
`fieldnames`	Field names of a structure
`getfield`	Get field of structure array
`rmfield`	Remove structure fields
`setfield`	Set field of structure array
`struct`	Create structure array
`struct2cell`	Structure to cell array conversion

MATLAB Object Functions

Using the object functions you can create objects, detect objects of a given class, and return the class of an object.

Object Functions	
`class`	Create object or return class of object
`isa`	Detect an object of a given class
`methods`	Display method names
`methodsview`	Displays information on all methods implemented by a class
`subsasgn`	Overloaded method for A(I)=B, A{I}=B, and A.field=B
`subsindex`	Overloaded method for X(A)
`subsref`	Overloaded method for A(I), A{I} and A.field

MATLAB Interface to Java Functions

These functions allow you to bring Java classes into MATLAB, construct objects, and call and save methods.

Interface to Java Functions	
`class`	Create object or return class of object
`import`	Add a package or class to the current Java import list
`isa`	Detect an object of a given class
`isjava`	Test whether an object is a Java object
`javaArray`	Constructs a Java array
`javaMethod`	Invokes a Java method
`javaObject`	Constructs a Java object
`methods`	Display method names
`methodsview`	Display information on all methods imple.mented by a class

Cell Array Functions

Cell arrays are arrays comprised of cells, which can hold any MATLAB data type such as text, numeric arrays, or other cell arrays. Unlike structures, you access these cells by number. Use the cell array functions to create and operate on these arrays.

Cell Array Functions	
`cell`	Create cell array
`cellfun`	Apply a function to each element in a cell array
`cellstr`	Create cell array of strings from character array
`cell2struct`	Cell array to structure array conversion
`celldisp`	Display cell array contents
`cellplot`	Graphically display the structure of cell arrays
`num2cell`	Convert a numeric array into a cell array

Multidimensional Array Functions

These functions provide a mechanism for working with arrays of dimension greater than 2.

Multidimensional Array Functions	
`cat`	Concatenate arrays
`flipdim`	Flip array along a specified dimension
`ind2sub`	Subscripts from linear index
`ipermute`	Inverse permute the dimensions of a multidimensional array
`ndgrid`	Generate arrays for multidimensional functions and interpolation
`ndims`	Number of array dimensions

Multidimensional Array Functions (Continued)	
`permute`	Rearrange the dimensions of a multidimensional array
`reshape`	Reshape array
`shiftdim`	Shift dimensions
`squeeze`	Remove singleton dimensions
`sub2ind`	Single index from subscripts

Data Visualization

This extensive set of functions gives you the ability to create basic graphs such as bar, pie, polar, and three-dimensional plots, and advanced graphs such as surface, mesh, contour, and volume visualization plots. In addition, you can use these functions to control lighting, color, view, and many other fine manipulations.

Basic Plots and Graphs	
`bar`	Vertical bar chart
`barh`	Horizontal bar chart
`hist`	Plot histograms
`histc`	Histogram count
`hold`	Hold current graph
`loglog`	Plot using log-log scales
`pie`	Pie plot
`plot`	Plot vectors or matrices.
`polar`	Polar coordinate plot
`semilogx`	Semi-log scale plot
`semilogy`	Semi-log scale plot
`subplot`	Create axes in tiled positions

Three-Dimensional Plotting	
`bar3`	Vertical 3-D bar chart
`bar3h`	Horizontal 3-D bar chart
`comet3`	3-D comet plot
`cylinder`	Generate cylinder

Three-Dimensional Plotting (Continued)	
fill3	Draw filled 3-D polygons in 3-space
plot3	Plot lines and points in 3-D space
quiver3	Three-dimensional quiver (or velocity) plot
slice	Volumetric slice plot
sphere	Generate sphere
stem3	Plot discrete surface data
waterfall	Waterfall plot

Plot Annotation and Grids	
clabel	Add contour labels to a contour plot
datetick	Date formatted tick labels
grid	Grid lines for 2-D and 3-D plots
gtext	Place text on a 2-D graph using a mouse
legend	Graph legend for lines and patches
plotedit	Start plot edit mode to edit and annotate plots
plotyy	Plot graphs with Y tick labels on the left and right
title	Titles for 2-D and 3-D plots
xlabel	X-axis labels for 2-D and 3-D plots
ylabel	Y-axis labels for 2-D and 3-D plots
zlabel	Z-axis labels for 3-D plots

Surface, Mesh, and Contour Plots	
contour	Contour (level curves) plot
contourc	Contour computation
contourf	Filled contour plot
hidden	Mesh hidden line removal mode
meshc	Combination mesh/contourplot

Surface, Mesh, and Contour Plots (Continued)	
mesh	3-D mesh with reference plane
peaks	A sample function of two variables
surf	3-D shaded surface graph
surface	Create surface low-level objects
surfc	Combination surf/contourplot
surfl	3-D shaded surface with lighting
trimesh	Triangular mesh plot
trisurf	Triangular surface plot

Volume Visualization	
coneplot	Plot velocity vectors as cones in 3-D vector field
contourslice	Draw contours in volume slice plane
curl	Compute the curl and angular velocity of a vector field
divergence	Compute the divergence of a vector field
flow	Generate scalar volume data
interpstreamspeed	Interpolate streamline vertices from vector-field magnitudes
isocaps	Compute isosurface end-cap geometry
isocolors	Compute the colors of isosurface vertices
isonormals	Compute normals of isosurface vertices
isosurface	Extract isosurface data from volume data
reducepatch	Reduce the number of patch faces
reducevolume	Reduce number of elements in volume data set
shrinkfaces	Reduce the size of patch faces
slice	Draw slice planes in volume
smooth3	Smooth 3-D data

Volume Visualization (Continued)	
stream2	Compute 2-D stream line data
stream3	Compute 3-D stream line data
streamline	Draw stream lines from 2- or 3-D vector data
streamparticles	Draw stream particles from vector volume data
streamribbon	Draw stream ribbons from vector volume data
streamslice	Draw well-spaced stream lines from vector volume data
streamtube	Draw stream tubes from vector volume data
surf2patch	Convert surface data to patch data
subvolume	Extract subset of volume data set

Domain Generation	
griddata	Data gridding and surface fitting
meshgrid	Generation of X and Y arrays for 3-D plots

Specialized Plotting	
area	Area plot
box	Axis box for 2-D and 3-D plots
comet	Comet plot
compass	Compass plot
convhull	Convex hull
delaunay	Delaunay triangulation
dsearch	Search Delaunay triangulation for nearest point
errorbar	Plot graph with error bars
ezcontour	Easy to use contour plotter
ezcontourf	Easy to use filled contour plotter
ezmesh	Easy to use 3-D mesh plotter

Specialized Plotting (Continued)	
ezmeshc	Easy to use combination mesh/ contour plotter
ezplot	Easy to use function plotter
ezplot3	Easy to use 3-D parametric curve plotter
ezpolar	Easy to use polar coordinate plotter
ezsurf	Easy to use 3-D colored surface plotter
ezsurfc	Easy to use combination surface/ contour plotter
feather	Feather plot
fill	Draw filled 2-D polygons
fplot	Plot a function
inpolygon	True for points inside a polygonal region
pareto	Pareto char
pcolor	Pseudocolor (checkerboard) plot
pie3	3-D pie plot
plotmatrix	Scatter plot matrix
polyarea	Area of polygon
quiver	Quiver (or velocity) plot
ribbon	Ribbon plot
rose	Plot rose or angle histogram
scatter	Scatter plot
scatter3	3-D scatter plot
stairs	Stairstep graph
stem	Plot discrete sequence data
tsearch	Search for enclosing Delaunay triangle
voronoi	Voronoi diagram

View Control	
camdolly	Move camera position and target
camlookat	View specific objects
camorbit	Orbit about camera target
campan	Rotate camera target about camera position
campos	Set or get camera position
camproj	Set or get projection type
camroll	Rotate camera about viewing axis
camtarget	Set or get camera target
camup	Set or get camera up-vector
camva	Set or get camera view angle
camzoom	Zoom camera in or out
daspect	Set or get data aspect ratio
pbaspect	Set or get plot box aspect ratio
view	3-D graph viewpoint specification.
viewmtx	Generate view transformation matrices
xlim	Set or get the current x-axis limits
ylim	Set or get the current y-axis limits
zlim	Set or get the current z-axis limits

Lighting	
camlight	Create or position a light
light	Light object creation function
lightangle	Spherical position of a light
lighting	Lighting mode
material	Material reflectance mode

Transparency	
alpha	Set or query transparency properties for objects in current axes
alphamap	Specify the figure alphamap
alim	Set or query the axes alpha limits

Color Operations	
brighten	Brighten or darken color map
caxis	Pseudocolor axis scaling
colorbar	Display color bar (color scale)
colordef	Set up color defaults
colormap	Set the color look-up table (list of colormaps)
graymon	Graphics figure defaults set for grayscale monitor
hsv2rgb	Hue-saturation-value to red-green-blue conversion
rgb2hsv	RGB to HSV conversion
rgbplot	Plot color map
shading	Color shading mode
spinmap	Spin the colormap
surfnorm	3-D surface normals
whitebg	Change axes background color for plots

Colormaps	
autumn	Shades of red and yellow color map
bone	Gray-scale with a tinge of blue color map
contrast	Gray color map to enhance image contrast
cool	Shades of cyan and magenta color map
copper	Linear copper-tone color map

Colormaps (Continued)	
flag	Alternating red, white, blue, and black color map
gray	Linear gray-scale color map
hot	Black-red-yellow-white color map
hsv	Hue-saturation-value (HSV) color map
jet	Variant of HSV
lines	Line color colormap
prism	Colormap of prism colors
spring	Shades of magenta and yellow color map
summer	Shades of green and yellow colormap
winter	Shades of blue and green color map

Printing	
orient	Hardcopy paper orientation
pagesetupdlg	Page position dialog box
print	Print graph or save graph to file
printdlg	Print dialog box
printopt	Configure local printer defaults
saveas	Save figure to graphic file

Handle Graphics, General	
allchild	Find all children of specified objects
copyobj	Make a copy of a graphics object and its children
findall	Find all graphics objects (including hidden handles)
findobj	Find objects with specified property values
gcbo	Return object whose callback is currently executing
gco	Return handle of current object

Handle Graphics, General (Continued)	
get	Get object properties
ishandle	True for graphics objects
rotate	Rotate objects about specified origin and direction
set	Set object properties

Working with Application Data	
getappdata	Get value of application data
isappdata	True if application data exists
rmappdata	Remove application data
setappdata	Specify application data

Handle Graphics, Object Creation	
axes	Create axes object
figure	Create figure (graph) windows
image	Create image (2-D matrix)
light	Create light object (illuminates patch and surface)
line	Create line object (3-D polylines)
patch	Create patch object (polygons)
rectangle	Create rectangle object (2-D rectangle)
surface	Create surface (quadrilaterals)
text	Create text object (character strings)
uicontextmenu	Create context menu (pop-up associated with object)

Handle Graphics, Figure Windows	
capture	Screen capture of the current figure
clc	Clear figure window
clf	Clear figure
close	Close specified window
closereq	Default close request function
gcf	Get current figure handle

Handle Graphics, Figure Windows (Continued)	
`newplot`	Graphics M-file preamble for `NextPlot` property
`refresh`	Refresh figure
`saveas`	Save figure or model to desired output format

Handle Graphics, Axes	
`axis`	Plot axis scaling and appearance
`cla`	Clear axes
`gca`	Get current axes handle

Object Manipulation	
`reset`	Reset axis or figure
`rotate3d`	Interactively rotate the view of a 3-D plot
`selectmoveresize`	Interactively select, move, or resize objects

Interactive User Input	
`ginput`	Graphical input from a mouse or cursor
`zoom`	Zoom in and out on a 2-D plot

Region of Interest	
`dragrect`	Drag XOR rectangles with mouse
`drawnow`	Complete any pending drawing
`rbbox`	Rubberband box

Graphical User Interfaces

The graphical user interface functions let you build your own interfaces for your applications.

Dialog Boxes	
`dialog`	Create a dialog box
`errordlg`	Create error dialog box
`helpdlg`	Display help dialog box
`inputdlg`	Create input dialog box
`listdlg`	Create list selection dialog box
`msgbox`	Create message dialog box
`pagedlg`	Display page layout dialog box
`printdlg`	Display print dialog box
`questdlg`	Create question dialog box
`uigetfile`	Display dialog box to retrieve name of file for reading
`uiputfile`	Display dialog box to retrieve name of file for writing
`uisetcolor`	Interactively set a `ColorSpec` using a dialog box
`uisetfont`	Interactively set a font using a dialog box
`warndlg`	Create warning dialog box

User Interface Deployment	
`guidata`	Store or retrieve application data
`guihandles`	Create a structure of handles
`movegui`	Move GUI figure onscreen
`openfig`	Open or raise GUI figure

User Interface Development	
`guide`	Open the GUI Layout Editor
`inspect`	Display Property Inspector

User Interface Objects	
menu	Generate a menu of choices for user input
uicontextmenu	Create context menu
uicontrol	Create user interface control
uimenu	Create user interface menu

Other Functions	
dragrect	Drag rectangles with mouse
findfigs	Display off-screen visible figure windows
gcbf	Return handle of figure containing callback object
gcbo	Return handle of object whose callback is executing
rbbox	Create rubberband box for area selection
selectmoveresize	Select, move, resize, or copy axes and uicontrol graphics objects
textwrap	Return wrapped string matrix for given uicontrol
uiresume	Used with `uiwait`, controls program execution
uiwait	Used with `uiresume`, controls program execution
waitbar	Display wait bar
waitforbuttonpress	Wait for key/buttonpress over figure

Serial Port I/O

These functions provides direct access to peripheral devices that you connect to your computer's serial port.

Creating a Serial Port Object	
serial	Create a serial port object

Writing and Reading Data	
fgetl	Read one line of text from the device and discard the terminator
fgets	Read one line of text from the device and include the terminator
fprintf	Write text to the device
fread	Read binary data from the device
fscanf	Read data from the device, and format as text
fwrite	Write binary data to the device
readasync	Read data asynchronously from the device
stopasync	Stop asynchronous and write operations

Configuring and Returning Properties	
get	Return serial port object properties
set	Configure or display serial port object properties

State Change	
fclose	Disconnect a serial port object from the device
fopen	Connect a serial port object to the device
record	Record data and event information to a file

General Purpose	
`clear`	Remove a serial port object from the MATLAB workspace
`delete`	Remove a serial port object from memory
`disp`	Display serial port object summary information
`instraction`	Display event information when an event occurs
`instrfind`	Return serial port objects from memory to the MATLAB workspace
`isvalid`	Determine if serial port objects are valid
`length`	Length of serial port object array
`load`	Load serial port objects and variables into the MATLAB workspace
`save`	Save serial port objects and variables to a MAT-file
`serialbreak`	Send a break to the device connected to the serial port
`size`	Size of serial port object array

Symbolic Math Toolbox Quick Reference

Introduction

This appendix lists the Symbolic Math Toolbox functions that are available in the Student Version of MATLAB & Simulink. For complete information about any of these functions, use Help and select Reference from the **Symbolic Math Toolbox**.

Note All of the functions listed in Symbolic Math Toolbox Reference are available in the Student Version of MATLAB & Simulink except `maple`, `mapleinit`, `mfun`, `mfunlist`, and `mhelp`.

Arithmetic Operations	
+	Addition
-	Subtraction
*	Multiplication
.*	Array multiplication
/	Right division
./	Array right division
\	Left division
.\	Array left division
^	Matrix or scalar raised to a power
.^	Array raised to a power
'	Complex conjugate transpose
.'	Real transpose

Basic Operations	
`ccode`	C code representation of a symbolic expression
`conj`	Complex conjugate
`findsym`	Determine symbolic variables
`fortran`	Fortran representation of a symbolic expression
`imag`	Imaginary part of a complex number
`latex`	LaTeX representation of a symbolic expression
`pretty`	Pretty print a symbolic expression
`real`	Real part of an imaginary number
`sym`	Create symbolic object
`syms`	Shortcut for creating multiple symbolic objects

Calculus	
`diff`	Differentiate
`int`	Integrate
`jacobian`	Jacobian matrix
`limit`	Limit of an expression

Calculus (Continued)	
`symsum`	Summation of series
`taylor`	Taylor series expansion

Conversions	
`char`	Convert `sym` object to string
`double`	Convert symbolic matrix to `double`
`poly2sym`	Function calculator
`sym2poly`	Symbolic polynomial to coefficient vector

Integral Transforms	
`fourier`	Fourier transform
`ifourier`	Inverse Fourier transform
`ilaplace`	Inverse Laplace transform
`iztrans`	Inverse z-transform
`laplace`	Laplace transform
`ztrans`	z-transform

Linear Algebra	
`colspace`	Basis for column space
`det`	Determinant
`diag`	Create or extract diagonals
`eig`	Eigenvalues and eigenvectors
`expm`	Matrix exponential
`inv`	Matrix inverse
`jordan`	Jordan canonical form
`null`	Basis for null space
`poly`	Characteristic polynomial
`rank`	Matrix rank
`rref`	Reduced row echelon form
`svd`	Singular value decomposition
`tril`	Lower triangle
`triu`	Upper triangle

Pedagogical and Graphical Applications	
ezcontour	Contour plotter
ezcontourf	Filled contour plotter
ezmesh	Mesh plotter
ezmeshc	Combined mesh and contour plotter
ezplot	Function plotter
ezplot	Easy-to-use function plotter
ezplot3	3-D curve plotter
ezpolar	Polar coordinate plotter
ezsurf	Surface plotter
ezsurfc	Combined surface and contour plotter
funtool	Function calculator
rsums	Riemann sums
taylortool	Taylor series calculator

Simplification	
collect	Collect common terms
expand	Expand polynomials and elementary functions
factor	Factor
horner	Nested polynomial representation
numden	Numerator and denominator
simple	Search for shortest form
simplify	Simplification
subexpr	Rewrite in terms of subexpressions

Solution of Equations	
compose	Functional composition
dsolve	Solution of differential equations
finverse	Functional inverse
solve	Solution of algebraic equations

Special Functions	
cosint	Cosine integral, *Ci(x)*
lambertw	Solution of $\lambda(x)e^{\lambda(x)} = x$
sinint	Sine integral, *Si(x)*
zeta	Riemann `zeta` function

Variable Precision Arithmetic	
digits	Set variable precision accuracy
vpa	Variable precision arithmetic

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Symbols